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THE PERFECT GUIDE

MATHEMATICS - IIA

THEORY OF EQUATIONS

2M

1) Solve the equation $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$.

Given Polynomial $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$ — (1)

Given polynomial is reciprocal of class two then -1 is one root.

$$\begin{array}{r|rrrrrr} -1 & 2 & 1 & -12 & -12 & 1 & 2 \\ & 0 & -2 & 1 & 11 & 1 & -2 \\ \hline & 2 & -1 & -11 & -1 & 2 & \boxed{0} \end{array}$$

$\therefore f(x) = 2x^4 - x^3 - 11x^2 - x + 2 = 0$.

Divide with x^2 on b.s.

$2x^2 - x - 11 - \frac{1}{x} + \frac{2}{x^2} = 0$.

$$2\left(x^2+\frac{1}{x^2}\right)-1\left(x+\frac{1}{x}\right)-11=0 \quad -(2)$$

Let $x+\frac{1}{x}=t$.

S.O.B.S.

$$x^2+\frac{1}{x^2}+2=t^2$$

$$x^2+\frac{1}{x^2}=t^2-2$$

Substitute the above values in eqn (2)

$$2(t^2-2)-1(t)-11=0.$$

$$2t^2-4-t-11=0.$$

$$2t^2-t-15=0$$

$$2t^2-6t+5t-15=0.$$

$$2t(t-3)+5(t-3)=0$$

$$(t-3)(2t+5)=0.$$

Case (i): $t=3$

$$x+\frac{1}{x}=3$$

$$x^2+1=3x$$

$$x^2-3x+1=0.$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9-4}}{2}$$

Case (ii): $2t=-5$

$$t=-5/2$$

$$x^2+1=\frac{5x}{2}$$

$$2x^2+2-5x=0.$$

$$2x^2-4x-x+2=0.$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$2x(x-2) - 1(x-2) = 0$$

$$(x-2)(2x-1) = 0$$

$$x = 2 \qquad 2x = 1$$

$$\Rightarrow \boxed{x = \tfrac{1}{2}}$$

$\therefore$ The roots are $-1,\ 2,\ \frac{1}{2},\ \frac{3 \pm \sqrt{5}}{2}$.

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2) Given that the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in

i) A.P. Show that $2p^3 - 3pq + r = 0$

ii) G.P. show that $p^3 r = q^3$

iii) H.P. show that $2q^3 = r(3pq - r)$

3 terms are in A.P

$a-d, a, a+d$

3 terms are in G.P

$\frac{a}{r}, a, ar.$

Sol: Given equation $x^3+3px^2+3qx+r=0$.

i) Let the roots are in A.P. i.e $a-d, a, a+d$ =

$$S_1 = a-\cancel{d}+a+a+\cancel{d} = -\frac{3p}{1}$$

$$3a = -3p.$$

$$\boxed{a=-p}$$

Here a is one root of given Polynomial i.e $\boxed{f(a)=0}$

$f(a) = f(-p) = (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0.$

$-p^3 + 3p^3 - 3qp + r = 0.$

$2p^3 - 3qp + r = 0.$

(ii) Let the roots are in G.P.

i.e $\frac{a}{r}, a, ar.$

$S_3 = \left(\frac{a}{\cancel{r}}\right) \cdot a \cdot (a\cancel{r}) = -\frac{r}{1}$

$a^3 = -r$

$a = -(r)^{1/3}$

$f(a) = 0$

$f(-r^{1/3}) = (-r^{1/3})^3 + 3p(-r^{1/3})^2 + 3q(-r^{1/3}) + r = 0$

$= -\cancel{r} - 3pr^{2/3} - 3qr^{1/3} + \cancel{r} = 0$

$\cancel{3}pr^{2/3} = \cancel{3}qr^{1/3}$

C.O.B.S

$$\boxed{p^3 r^2 = q^3 r}$$

(iii) from case(i) $a = -p \Rightarrow \frac{1}{a} = -\frac{1}{p}.$

$f(1/a) = f(-1/p) = \left(-\frac{1}{p}\right)^3 + 3p\left(-\frac{1}{p}\right)^2 + 3q\left(-\frac{1}{p}\right) + r = 0.$

$-\frac{1}{p^3} + \frac{3p}{p^2} - \frac{3q}{p} + r = 0.$

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3) Solve $18x^3 + 81x^2 + 121x + 60 = 0$, given that one root is equal to half the sum of the remaining roots.

Sol. Given equation $18x^3+81x^2+121x+60=0$ — (1)

Let α, β, γ are the roots of equation (1)

one root is equal to half the sum of the remaining roots.

$$\gamma = \frac{\alpha+\beta}{2} \text{ (or) } \alpha = \frac{\beta+\gamma}{2} \text{ (or) } \beta = \frac{\alpha+\gamma}{2}$$

$$2\beta = \alpha+\gamma$$

$$S_1 = \alpha+\beta+\gamma = \frac{-81^{9}}{18_{2}} \Rightarrow \alpha+\gamma+\beta = -9/2$$

$$2\beta+\beta = -9/2$$

$$3\beta = -9/2$$

$$\boxed{\beta = -3/2}$$

$\beta = -\frac{3}{2}$

$2\beta = \alpha + \gamma$

$-\cancel{2}\left(\frac{3}{\cancel{2}}\right) = \alpha + \gamma$

$\boxed{\alpha + \gamma = -3}$ — (2)

$S_3 = \alpha\beta\gamma = -\frac{60}{18}$

$\alpha\gamma\left(+\frac{\cancel{3}}{\cancel{2}}\right) = +\frac{\cancel{60}\,20}{\cancel{18}\,9}$

$\boxed{\alpha\gamma = +\frac{20}{9}}$

$\alpha - \gamma = \sqrt{(-3)^2 - 4\left(\frac{20}{9}\right)}$

$\alpha - \gamma = \sqrt{9 - \frac{80}{9}}$

$\alpha - \gamma = \sqrt{\frac{81-80}{9}} = \sqrt{\frac{1}{9}}$

$\alpha - \gamma = \pm\frac{1}{3} \Rightarrow \alpha - \gamma = \frac{1}{3}$ & $\alpha - \gamma = -\frac{1}{3}$

$\alpha + \gamma = -3$

$\alpha - \gamma = \frac{1}{3}$

$2\alpha = -3 + \frac{1}{3}$

$2\alpha = -\frac{8}{3}$

$\boxed{\alpha = -\frac{4}{3}}$

Substitute α in (2).

$-\frac{4}{3} + \gamma = -3$

$\gamma = -3 + \frac{4}{3}$

$\boxed{\gamma = -\frac{5}{3}}$

$(a-b)^2 = (a+b)^2 - 4ab$

$(a-b) = \sqrt{(a+b)^2 - 4ab}$

$\therefore$ The roots are

$-\frac{4}{3}, -\frac{3}{2}, -\frac{5}{3}.$

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4) Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two roots is 6.

Sol: Given equation $x^4+x^3-16x^2-4x+48=0$

Let α, β, γ and δ are the roots

Given that the product of two roots is 6. i.e $\boxed{\alpha\beta=6}$

$S_1 = \alpha+\beta+\gamma+\delta = -\frac{1}{1} = -1$

$S_4 = \alpha\beta\gamma\delta = -48$

$6\gamma\delta = -48$

$\boxed{\gamma\delta=-8}$

$S_3 = \alpha\beta\gamma + \alpha\beta\delta + \beta\gamma\delta + \gamma\alpha\delta = -4$

$6\gamma + 6\delta + -8\beta - 8\alpha = -4$

$6(\gamma+\delta) - 8(\beta+\alpha) = -4$

$$6(\gamma+\delta)-8(\alpha+\beta)=-4$$

from S_1 $\boxed{\alpha+\beta=-1-(\gamma+\delta)}$

$$6(\gamma+\delta)-8\left[-1-(\gamma+\delta)\right]=-4$$

$$6(\gamma+\delta)+8+8(\gamma+\delta)=-4$$

$$14(\gamma+\delta)=-12$$

$$\boxed{\gamma+\delta=\frac{-12}{14}}$$

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5) Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in the A.P.

Sol. Given equation $4x^3 - 24x^2 + 23x + 18 = 0$

Given the roots are in A.P.

i.e $a-d, a, a+d$

$S_1 = a-d+a+a+d = -\frac{(-24)}{4}$

$3a = 6$

$\boxed{a = 2}$

$S_3 = (a-d)(a)(a+d) = -\frac{18}{4}$

$2(a^2-d^2) = -\frac{18}{4}$

$4 - d^2 = -\frac{9}{4}$

$4 + \frac{9}{4} = d^2$

$\frac{16+9}{4} = d^2$

$d^2 = \frac{25}{4}$

$d = \pm \frac{25}{4} \Rightarrow d = \frac{25}{4} \quad d = -\frac{25}{4}$

$\boxed{a = 2}$

$\Rightarrow a - d = 2 - \frac{25}{4} = \frac{8-25}{4}$

$= -17/4$

$\Rightarrow a + d = 2 + \frac{25}{4} = \frac{8+25}{4} = \frac{33}{4}$

$\therefore$ The roots are $-\frac{17}{4}, 2, \frac{33}{4}$.

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6) Solve the equation $x^3 - 7x^2 + 14x - 8 = 0$, given that the roots are in G.P.

Sol: Given equation $x^3 - 7x^2 + 14x - 8 = 0$

And the roots are in G.P

i.e $\frac{a}{r}, a, ar.$

$S_3 = \left(\frac{a}{\cancel{r}}\right)(a)(a\cancel{r}) = \frac{-(-8)}{1}$

$a^3 = 8$

$a^3 = 2^3$

$\boxed{a = 2}$

$S_1 = \frac{a}{r} + a + ar = \frac{-(-7)}{1}$

$\frac{a + ar + ar^2}{r} = 7.$

$2 + 2r + 2r^2 = 7r$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

$2r(r-2) - 1(r-2) = 0$

$(r-2)(2r-1) = 0$

Case (i)

$\boxed{r = 2}$

(i) $a = 2$

(ii) $\frac{a}{r} = \frac{2}{r} = 1$

(iii) $a \cdot r = 2 \cdot 2 = 4$

Case (ii): $2r = 1$

$\boxed{r = \frac{1}{2}}$

(i) $a = 2$

(ii) $\frac{a}{r} = \frac{2}{\frac{1}{2}} = 4$

(iii) $a \cdot r = 2 \cdot \frac{1}{2} = 1$

$\therefore$ The roots are 1, 2, 4.

Jm.

7) Solve $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$, given that it has two pairs of equal roots

Sol: Given equation $x^4+4x^3-2x^2-12x+9=0$

It has two pairs of equal roots i.e (α,α) (β,β)

$S_1 = \alpha+\beta+\gamma+\delta = -\frac{4}{1}$

$2\alpha+2\beta = -4$

$\boxed{\alpha+\beta=-4}$ —(1)

$S_4 = \alpha\beta(\alpha\beta) = 9$

$\alpha^2\beta^2 = 9.$

$\boxed{\alpha\beta = \pm 3}$

$\boxed{\alpha\beta = 3}$ $\boxed{\alpha\beta = -3}$

Case (i)

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta.$$

$$(\alpha-\beta) = \sqrt{(-4)^2 - 4(3)}$$

$$\alpha-\beta = \sqrt{16-12}$$

$$\boxed{\alpha-\beta = 2} \quad -(2)$$

Solve (1) & (2).

$$\alpha+\beta = -4$$

$$\alpha-\beta = 2$$

$$2\alpha = -2$$

$$\boxed{\alpha = -1}$$

Substitute $\alpha = -1$ in eqn (1).

$$-1+\beta = -4$$

$$\boxed{\beta = -3}$$

Case (ii)

$$\alpha\beta = -3$$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta.$$

$$(\alpha-\beta) = \sqrt{(-4)^2 - 4(-3)}$$

$$(\alpha-\beta) = \sqrt{16+12}$$

$$\alpha-\beta = \sqrt{28}$$

$$\boxed{\alpha-\beta = 2\sqrt{7}} \quad -(3)$$

Solve (1) & (3)

$$\alpha+\beta = -4$$

$$\alpha-\beta = 2\sqrt{7}$$

$$2\alpha = -4+2\sqrt{7}$$

$$\boxed{\alpha = -2+\sqrt{7}}$$

Substitute α in eqn (1).

$$-2+\sqrt{7}+\beta = -4$$

$$\boxed{\beta = -2-\sqrt{7}}$$

$\therefore$ The roots are $(-1,-3)$, $(-1,-3)$ & $(-2+\sqrt{7}, -2-\sqrt{7})$, $(-2+\sqrt{7}, -2-\sqrt{7})$

8) Find the polynomial equation of degree 5 whose roots are the translate of the roots of $x^5 + 4x^3 - x^2 + 11 = 0$, by -3.

Note: $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots + a_n = 0$.

$f(x+h) = A_0x^n + A_1x^{n-1} + A_2x^{n-2} + A_3x^{n-3} + \cdots + A_n = 0$.

Sol: Given equation $f(x) = x^5 + 4x^3 - x^2 + 11 = 0$.

-3 is the root of the Polynomial then

Transformed Polynomial is $f(x) = A_0x^5 + A_1x^4 + A_2x^3 + A_3x^2 + A_4x + A_5 = 0$

-3	1	0	4	-1	0	11	
	0	-3	9	-39	120	-360	
-3	1	-3	13	-40	120	$\boxed{-349} = A_5$	
	0	-3	18	-93	399		
-3	1	-6	31	-133	$\boxed{519} = A_4$		
	0	-3	27	-174			
-3	1	-9	58	$\boxed{-307} = A_3$			
	0	-3	36				
-3	1	-12	$\boxed{94} = A_2$				
	0	-3					
-3	1	$\boxed{-15} = A_1$					
	0						
	$1 = A_0$						

$\therefore$ The transformed equation is

$$f(x) = x^5 - 15x^4 + 94x^3 - 307x^2 + 519x - 349 = 0$$

$1 \times 7 = 07$

$1 \times 4 = 04$

$1 \times 2 = 02$

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THANK YOU