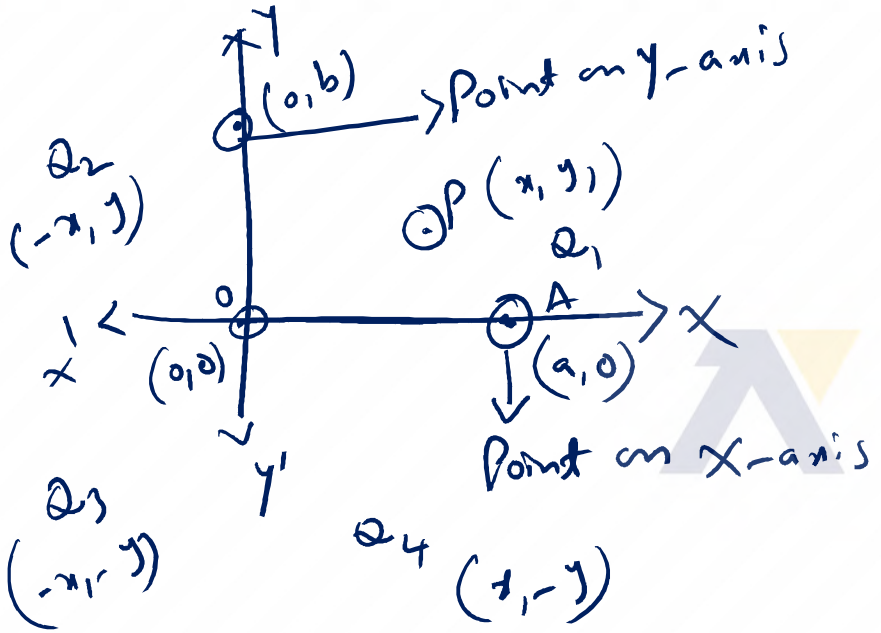
MATHEMATICS - IB

STRAIGHT LINES

General equation $ax+by+c=0$



$A(x_1, y_1)$ $B(x_2, y_2)$

distance between $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$A(x_1, y_1)$ $B(x_2, y_2)$ slope

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$m = \tan\theta$ → in terms of Angle

The equation of a line in two points form.

$$(y-y_1) = m(x-x_1)$$

Here $m = \frac{y_2-y_1}{x_2-x_1}$

(or)

$$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$$

2M

1) Find the value of x, if the slope of the line joining (2, 5), (x, 3) is 2.

Sol. Given Points $(2, 5)$ $(x, 3)$
x_1, y_1 x_2, y_2

Slope $(m) = 2$

$\therefore$ Slope $(m) = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{3-5}{x-2}$

$2 = \frac{-2}{x-2}$

$x - 2 = -1$

$\boxed{x = 1}$

2m

2) Find the equation of the line passing through the point (3, -2) and having inclination 135° (θ)

Sol: Given point $(3, -2)$

Angle $(\theta) = 135^\circ$

$m = \tan\theta$

$m = \tan 135^\circ$

$m = \tan(90+45^\circ)$

$m = -\cot 45^\circ$

$\boxed{m = -1}$

∴ The equation of a line is

$(y - y_1) = m(x - x_1)$

$[y - (-2)] = -1[x - 3]$

$y + 2 = -x + 3$

$\boxed{x + y - 1 = 0}$

2M

3) Find the equation of the line passing through the points $(at_1^2, 2at_1), (at_2^2, 2at_2)$

Sol. Given points $(at_1^2, 2at_1)$ $(at_2^2, 2at_2)$

x_1 y_1 x_2 y_2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$$

$$= \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2(t_2 - t_1)}{(t_2 - t_1)(t_2 + t_1)}$$

$$m = \frac{2}{t_2 + t_1}$$

∴ The equation of a line is

$$(y - 2at_1) = \frac{2}{t_2 + t_1}(x - at_1^2)$$

$$y(t_2 + t_1) - 2at_1t_2 - 2at_1^2 = 2x - 2at_1^2$$

$$\therefore 2x + 2at_1t_2 = y(t_2 + t_1)$$

4) The area of the triangle formed by the coordinate axes and the line $4x - 3y = 12$ is

Note: If $ax+by+c=0 \Rightarrow$ Area of a $\Delta^{le} = \frac{c^2}{2|ab|}$ sq. units.

Sol: Given line $4x - 3y - 12 = 0$

Here $a=4 \quad b=-3 \quad c=-12$

$\therefore$ Area of a triangle $= \frac{(-12)^2}{2|4(-3)|} = \frac{144}{24}$

$= 6$ sq. units

5) The area of the triangle formed by the coordinate axes and the line $\frac{x}{2} + \frac{y}{3} = 1$ is

Note: $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow$ Area of a $\Delta^{le} = \frac{1}{2}|ab|$ sq. units

Sol: Given line $\frac{x}{2} + \frac{y}{3} = 1$

Here $a = 2$ $b = 3$ $\left(\because \frac{x}{a} + \frac{y}{b} = 1\right)$

$\therefore$ Area of a $\Delta^{le} = \frac{1}{2}|2 \times 3|$

$= 3$ sq. units.

2M/4M

6) If the product of the intercepts made by the straight line $x \tan\alpha + y \sec\alpha = 1 (0 \le \alpha < \pi/2)$ on the coordinate axes is equal to $\sin\alpha$, find α

Sol: Given line $x\tan\alpha + y\sec\alpha = 1$

$$\frac{x}{\cot\alpha} + \frac{y}{\cos\alpha} = 1$$

Given Product of Intercepts is equal to $\sin\alpha$

i.e $\cot\alpha \cdot \cos\alpha = \sin\alpha$.

$$\frac{\cos\alpha}{\sin\alpha} \cdot \cos\alpha = \sin\alpha$$

$\cos^2\alpha = \sin^2\alpha$

$\Rightarrow \tan^2\alpha = 1$

$\tan\alpha = \pm 1$

$\tan\alpha = 1$	$\tan\alpha = -1$ (Q_2, Q_4)
$\tan\alpha = \tan\frac{\pi}{4}$	$\tan\alpha = \tan\frac{3\pi}{4}$
$\alpha = \frac{\pi}{4}$	$\alpha = \frac{3\pi}{4}$

$\therefore \alpha = \frac{\pi}{4}$ (or) 45° $\left(\because 0 \le \alpha < \frac{\pi}{2}\right)$

4M / 2M

intercepts equal sum of the intercept zero.

7) Find the equation of the line passing through the point (3, -4) and making x and y – intercepts which are in the ratio is 2 : 3

Sol. Given point $(3,-4)$

let $\frac{x}{a}+\frac{y}{b}=1$ — (1)

Given x and y- intercepts are in the ratio 2 : 3

i.e $\frac{a}{b}=\frac{2}{3} \Rightarrow a=\frac{2b}{3}$

from (1) $\frac{x}{2b/3}+\frac{y}{b}=1 \Rightarrow \frac{3x}{2b}+\frac{y}{b}=1$ — (2)

But equation (2) passes through $(3, -4)$

$$\frac{3(3)}{2b} + \frac{(-4)}{b} = 1$$

$$\Rightarrow \frac{9-8}{2b} = 1$$

$$\frac{1}{2b} = 1$$

$$2b = 1$$

$$\boxed{b = 1/2}$$

$$\therefore a = \frac{2b}{3} = \frac{2}{3}(1/2) \Rightarrow \boxed{a = 1/3}$$

Substitute a, b values in equation (1)

$$\frac{x}{1/3} + \frac{y}{1/2} = 1$$

$$3x + 2y = 1$$

(or)

$$3x + 2y - 1 = 0$$

4M

8) Transform the equation $\frac{x}{a} + \frac{y}{b} = 1$ into normal from where a > 0 and b > 0. If the perpendicular distance of the straight line from the origin is p, deduce that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Normal form

$x\cos\alpha + y\sin\alpha = p$

Sol: Given $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{bx+ay}{ab} = 1$

$bx + ay - ab = 0$

$bx + ay = ab$

divide with $\sqrt{b^2+a^2}$ on b.s

$\frac{bx}{\sqrt{a^2+b^2}} + \frac{ay}{\sqrt{a^2+b^2}} = \frac{ab}{\sqrt{a^2+b^2}}$

$x\cos\alpha + y\sin\alpha = p$

Here $\cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$

$\sin\alpha = \frac{a}{\sqrt{a^2+b^2}}$

$p = \frac{ab}{\sqrt{a^2+b^2}}$

[The Perpendicular distance from origin to line $ax+by+C=0$ is

$$d = \frac{|C|}{\sqrt{a^2+b^2}}$$]

$$p = \frac{|-ab|}{\sqrt{b^2+a^2}}$$

S.O.B.S

$$p^2 = \frac{a^2b^2}{a^2+b^2}$$

$$p^2(a^2+b^2) = a^2b^2$$

$$\frac{a^2+b^2}{a^2b^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$$\therefore \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence Proved

9) Transform the following equations into slope intercept form, intercepts form and normal form $x + y + 2 = 0$

Sol: Given line $x+y+2=0$

(i) Slope intercept form $(y=mx+c)$:

$x+y+2=0$

$y=-x-2$

$y=x(-1)-2$

Here $\boxed{m=-1}$

(ii) Intercept form $\left(\frac{x}{a}+\frac{y}{b}=1\right)$

$x+y=-2$

$\frac{x}{-2}+\frac{y}{-2}=\frac{-2}{-2}$

$\frac{x}{-2}+\frac{y}{-2}=1$

Here x-intercept $(a)=-2$

y-intercept $(b)=-2$

(iii) Normal form (or) Perpendicular form ($x\cos\alpha + y\sin\alpha = p$):

Given line $x+y+2=0$.

$x+y=-2$

$-x-y=2$

Divide with $\sqrt{2}$ on b.s.

$\frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{2}{\sqrt{2}}$ $\left(\because 2=\sqrt{2}\sqrt{2}\right)$

$x\left(\frac{-1}{\sqrt{2}}\right) + y\left(\frac{-1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

$x\cos\left(\pi+\frac{\pi}{4}\right) + y\sin\left(\pi+\frac{\pi}{4}\right) = p$

Q_3

Cos, Sin -ve

$\pi+\theta$, $\frac{3\pi}{2}-\theta$.

$x\cos\left(\frac{5\pi}{4}\right) + y\sin\left(\frac{5\pi}{4}\right) = p$

Here $\alpha = \frac{5\pi}{4}$

$p = \frac{1}{\sqrt{2}}$

JM

10) If the straight lines x + p = 0, y + 2 = 0 and 3x + 2y + 5 = 0 are concurrent, find the value of p.

If $a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

$a_3x+b_3y+c_3=0$

are concurrent.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

Sol: Given line $x+p=0$

$y+2=0$

$3x+2y+5=0$ are concurrent.

$$\begin{vmatrix} 1 & 0 & p \\ 0 & 1 & 2 \\ 3 & 2 & 5 \end{vmatrix} = 0 \Rightarrow$$

$$1\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 3 & 5 \end{vmatrix} + p\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} = 0$$

$1(5-4)+p(0-3)=0$

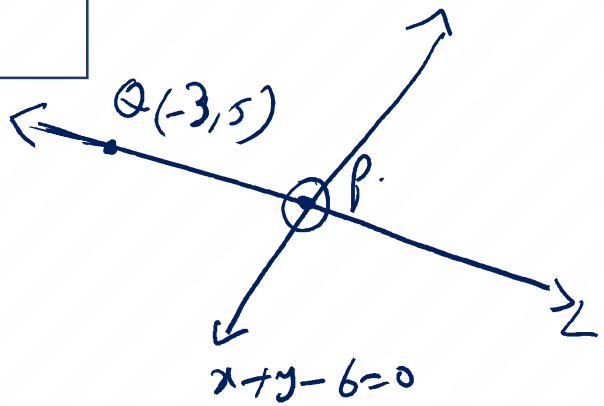
$1-3p=0$

$\Rightarrow 3p=1$

$$p = 1/3$$

4M

11) A straight line with slope 1 passes through Q(-3, 5) and meets the straight line x + y – 6 = 0 at P. Find the length PQ.



Sol: Given line $x+y-6=0$ — (1)

Point Q $(-3, 5)$ (x_1, y_1) slope $(m)=1$

∴ The equation of a line is

$(y-y_1)=m(x-x_1)$

$(y-5)=1(x+3)$

$x-y+3+5=0$

$x-y+8=0$ — (2)

Solve (1) & (2)

$x+y-6=0$

$x-y+8=0$

$2x+2=0$

$2x=-2$

$x=-1$

Substitute x in equation (1).

$-1 + y - 6 = 0$.

$\boxed{y = 7}$

$\therefore P$ is $(-1, 7)$ $\quad Q(-3, 5)$

$x_1\ y_1 \quad x_2\ y_2$

$\therefore$ length of $PQ = \sqrt{(-3-(-1))^2 + (5-7)^2}$

$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = \sqrt{4 \times 2}$

$\therefore PQ = 2\sqrt{2}$ units //

THANK YOU