Rizee®

THE PERFECT GUIDE

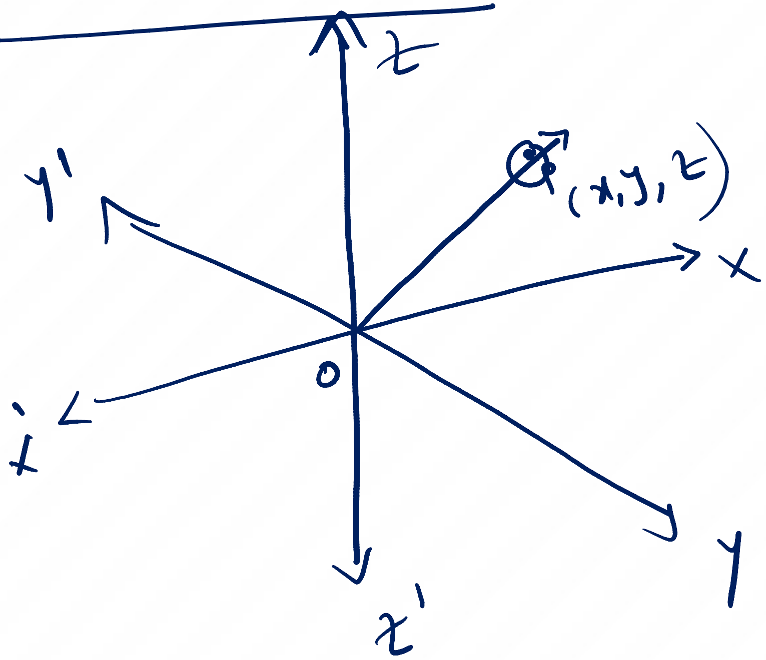
MATHEMATICS - IA

Addition of vectors

vector: magnitude, direction.

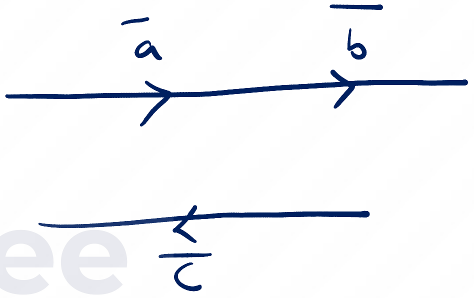
Scalar: magnitude.

Position of a vector:



unit vector: magnitude = 1 ⇒ $\boxed{\hat{a} = \frac{a}{|a|}}$

collinear, like vectors:



$\boxed{-\bar{c}}$

$\overline{OA} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\overline{OB} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\boxed{\overline{AB} = \overline{OB} - \overline{OA}}$

2M

1) Find unit vector in the direction of vector a = 2i + 3j + k.

Sol: Given vector $a = 2i + 3j + k$

$\therefore$ unitor vector $\hat{a} = \frac{a}{|a|}$ — (1)

$|a| = \sqrt{4+9+1}$

$|a| = \sqrt{14}$

from (1) $\hat{a} = \frac{2i+3j+k}{\sqrt{14}} = \left(\frac{2}{\sqrt{14}}\right)\bar{i} + \left(\frac{3}{\sqrt{14}}\right)\bar{j} + \left(\frac{1}{\sqrt{14}}\right)\bar{k}$

2) Find a vector in the direction of vector a = i – 2j that has magnitude 7 units.

Sol. Given vector $a = i - 2j$

$$|a| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

The vector having magnitude 7 units is

$$7a = 7\frac{a}{|a|} = \frac{7(i - 2j)}{\sqrt{5}} = \frac{7i - 14j}{\sqrt{5}}$$

3) Find the unit vector in the direction of the sum of the vectors.
a = 2i + 2j – 5k and b = 2i + j + 3k.

Sol. Given vector $a = 2i + 2j - 5k$

$b = 2i + j + 3k$

$a + b = 2i + 2j - 5k + 2i + j + 3k$

$a + b = 4i + 3j - 2k \Rightarrow |a+b| = \sqrt{16+9+4} = \sqrt{29}$

$\therefore$ unit vector $[\widehat{a+b}] = \dfrac{a+b}{|a+b|} = \dfrac{4i+3j-2k}{\sqrt{29}}$

$= \left(\dfrac{4}{\sqrt{29}}\right)i + \left(\dfrac{3}{\sqrt{29}}\right)j - \left(\dfrac{2}{\sqrt{29}}\right)k$

4) Let A B C D E F be a regular hexagon with centre 'O'. Show that AB + AC + AD + AE + AF = 3 AD = 6 AO.

Sol: Given $AB + AC + AD + AE + AF$

$= (AB + AE) + AD + (AC + AF)$

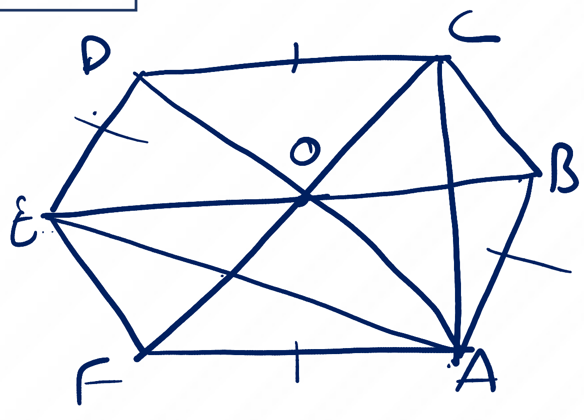
$= (ED + AE) + AD + (AC + CD)$ $\left[\because \begin{array}{l} AB = ED \\ AF = CD \end{array}\right]$

$= (\overline{OD} - \cancel{OE} + \cancel{OE} - OA) + AD + (\cancel{OC} - \overline{OA} + \overline{OD} - \cancel{OC})$

$= \overline{AD} + \overline{AD} + \overline{AD} = 3\overline{AD}$

$= 3(2\overline{AO}) = 6\overline{AO}$

Hence Proved



$\boxed{\overline{OA} + \overline{OD} = AD}$

$\boxed{2 \cdot AO = AD}$

5) In ΔABC, if O is the circumcentre and H is the orthocenter, then show that

(i) $OA + OB + OC = OH$ (ii) $HA + HB + HC = 2HO$

Sol. let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$

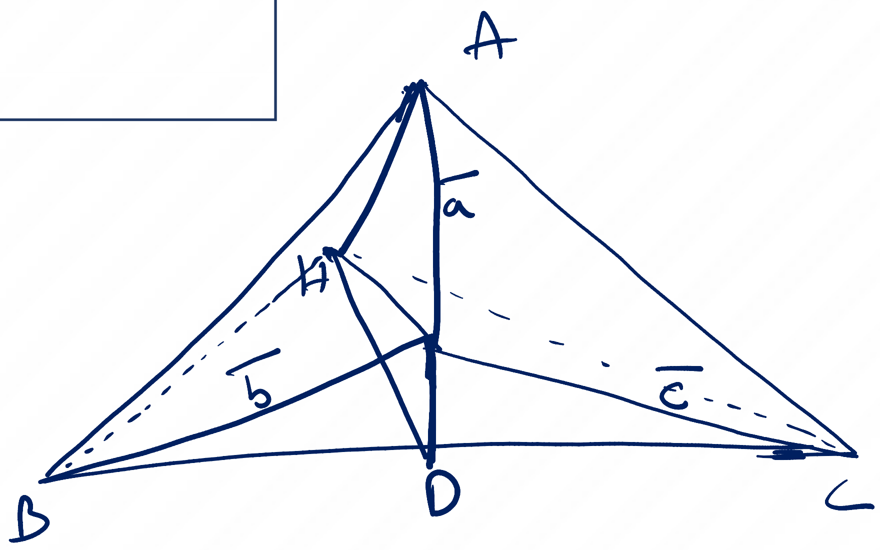
$$\overline{OD} = \frac{\bar{b}+\bar{c}}{2}$$

$$2\overline{OD} = \bar{b}+\bar{c}$$

$$OA + OB + OC = OA + 2\overline{OD} = OA + AH = OH$$

(R is circumradius of ΔABC)

Then $2 \cdot OD = AH$.



(ii) $\vec{HA} + \vec{HB} + \vec{HC} = \vec{HA} + 2\vec{HD}$.

$= \vec{HA} + 2(\vec{OD} - \vec{OH})$

$= \vec{HA} + 2\vec{OD} - 2\vec{OH}$

$= \vec{HA} + \vec{AH} - 2(-\vec{HO})$

$= \cancel{\vec{HA}} - \cancel{\vec{HA}} + 2\vec{HO}$

$= 2\vec{HO}$

Hence Proved

4M

6) If the points whose position vectors are

3i – 2j – k, 2i + 3j – 4k, -i + j + 2k and 4i + 5j + λk are coplanar, ~~then show~~ that

$\lambda = -\frac{146}{17}$.

Sol: Given vectors $\overline{OA} = 3i - 2j - k$ $\quad \overline{OC} = -i + j + 2k$

$\overline{OB} = 2i + 3j - 4k$ $\quad \overline{OD} = 4i + 5j + \lambda k$

$\overline{AB} = \overline{OB} - \overline{OA} = 2i + 3j - 4k - 3i + 2j + k = \underline{-\bar{i} + 5\bar{j} - 3\bar{k}}$

$\overline{AC} = \overline{OC} - \overline{OA} = -i + \bar{j} + 2\bar{k} - 3i + 2j + k = -4i + 3j + 3k$

$\overline{AD} = \overline{OD} - \overline{OA} = 4i + 5j + \lambda k - 3i + 2j + k = i + 7j + (\lambda + 1)k$

The given vectors are coplanar i.e

$$[\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$$

$$\begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0.$$

$$-1(3\lambda+3-21) - 5(-4\lambda-4-3) - 3(-28-3) = 0.$$

$$-1(3\lambda-18) - 5(-4\lambda-7) - 3(-31) = 0.$$

$$-3\lambda + 18 + 20\lambda + 35 + 93 = 0$$

$$17\lambda + 146 = 0.$$

$$17\lambda = -146$$

$$\boxed{\lambda = \frac{-146}{17}}$$

Hence Proved

2M

7) If the vectors -3i + 4j + λk and μi + 8j + 6k are collinear vectors, then find λ and μ.

Sol: Given vectors $-3i+4j+\lambda k$, $\mu i+8j+6k$

Given vectors are collinear i.e $\boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}$

$$\frac{-3}{\mu}=\frac{4}{8}=\frac{\lambda}{6}$$

$$\frac{-3}{\mu}=\frac{1}{2}$$

$$\boxed{\mu=-6}$$

$$\frac{1}{\cancel{2}_1}=\frac{\lambda}{\cancel{6}_3}$$

$$\boxed{\lambda=3}$$

8) a, b, c are non – coplanar vectors. Prove that then following four points are coplanar. –a + 4b - 3c, 3a + 2b – 5c, -3a + 8b – 5c, +3a + 2b + c .

Sol:

Given vectors $\overline{OA} = -a+4b-3c$ $\quad \overline{OC} = -3a+8b-5c$

$\overline{OB} = 3a+2b-5c$ $\quad \overline{OD} = -3a+2b+c$

$\overline{AB} = \overline{OB} - \overline{OA} = 3a+2b-5c+a-4b+3c = 4a-2b-2c$

$\overline{AC} = \overline{OC} - \overline{OA} = -3a+8b-5c+a-4b+3c = -2a+4b-2c$

$\overline{AD} = \overline{OD} - \overline{OA} = +3a+2b+c+a-4b+3c = +4a-2b+4c$

$\therefore [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0.$

$$\begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ +4 & -2 & 4 \end{vmatrix} = 0.$$

$$4(16-(4)) + 2(-8-(4)) - 2(4-(+8)) = 0.$$

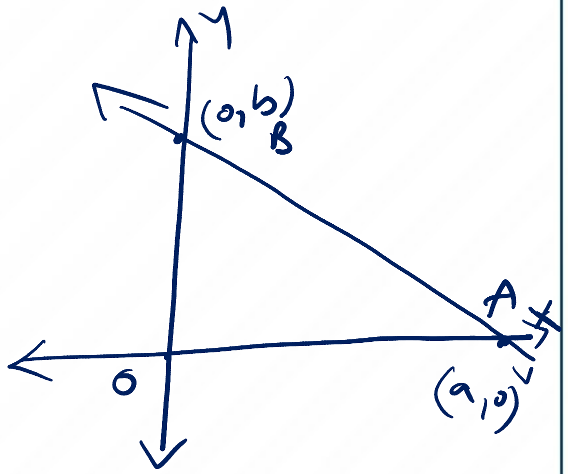
$$4(12) + 2(-12) - 2(-12) = 0.$$

$$48 - 24 + 24 \neq 0.$$

Hence The given vectors are non-coplanar.

4M

equation of the line whose intercepts on the axes are 'a' and 'b' is $\frac{x}{a}+\frac{y}{b}=$



Soln. Let $A=(a,0)$ $B=(0,b)$

$\overline{OA}=a\bar{i}$ $\overline{OB}=b\bar{j}$

$\therefore$ The line equation is $\bar{r}=(1-t)\overline{OA}+t\,\overline{OB}$

$=(1-t)a\bar{i}+tb\bar{j}$ — (1)

If $\bar{r}=x\bar{i}+y\bar{j}$ — (2)

from (1) & (2)

$(1-t)a=x$ | $tb=y$

$1-t=\frac{x}{a}$ | $t=\frac{y}{b}$

$$1 - t = \frac{x}{a}$$

$$\boxed{t = \frac{y}{b}.}$$

$$1 = \frac{x}{a} + t$$

$$\boxed{1 = \frac{x}{a} + \frac{y}{b}}$$

Hence Proved.

10) Find the equation of the line parallel to the vector 2i – j + 2k and which passes through the point A whose position vector is 3i + j – k. If P is a point on this line such that AP = 15, find the position vector of P.

Sol: The equation of a vector $\bar{r} = a + t\bar{b}$

$$\bar{r} = 3\hat{i} + \hat{j} - \hat{k} + t(2\hat{i} - \hat{j} + 2\hat{k}) \quad \text{--- (1)}$$

Now $\overline{AP} = t(2\hat{i} - \hat{j} + 2\hat{k})$

$$|\overline{AP}| = \sqrt{4t^2 + t^2 + 4t^2}$$

$$|\overline{AP}| = \sqrt{9t^2}$$

$$|\overline{AP}| = 3t.$$

$$AP = \pm 3t.$$

But Given $AP = 15$

$$15 = \pm 3t.$$

$$\therefore \boxed{t = \pm 5}$$

from (1)

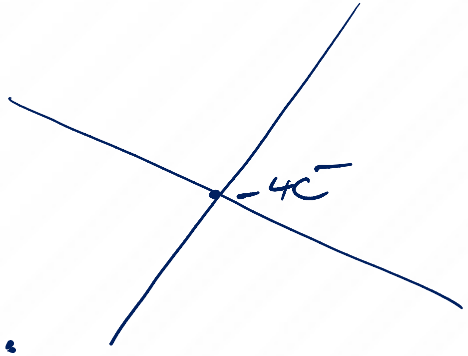
$$\bar{r} = (3\hat{i} + \hat{j} - 2\hat{k}) \pm 5(2\hat{i} - \hat{j} + 2\hat{k})$$

$$\bar{r} = 13\hat{i} - 4\hat{j} + 8\hat{k}$$

(or)

$$\bar{r} = -7\hat{i} + 6\hat{j} - 12\hat{k}$$

11) Show that the line joining the pair of points 6a – 4b + 4c, -4c and the line joining the pair of points –a – 2b – 3c, a + 2b – 5c intersect at the point -4c when a, b c are non – coplanar vectors



Sol: Given $6\bar{a}-4b+4c, -4c$ are represent one line.

$-a-2b-3c, a+2b-5c$ are represents another line.

∴ The equation of line $\bar{r} = (1-t)(6\bar{a}-4\bar{b}+4\bar{c}) + t(-4\bar{c})$

$\bar{r} = 6\bar{a}-4\bar{b}+4c-6t\bar{a}+4t\bar{b}-4t\bar{c}-4t\bar{c}$

$\bar{r} = \bar{a}(6-6t)+\bar{b}(4t-4)+\bar{c}(4-8t)$ — (1)

$\bar{r} = (1-s)(-a-2b-3\bar{c}) + s(a+2b-5c)$

$\bar{r} = -\bar{a} - 2b - 3\bar{c} + s\bar{a} + 2s\bar{b} + 3s\bar{c} + s\bar{a} + 2s\bar{b} - 5s\bar{c}$

$\bar{r} = -\bar{a} - 2\bar{b} - 3\bar{c} + 2s\bar{a} + 4s\bar{b} - 2s\bar{c}$

$\bar{r} = \bar{a}(2s-1) + \bar{b}(4s-2) + \bar{c}(-3-2s)$ — (2)

Now (1) = (2)

$6 - 6t = 2s - 1$

$2s + 6t + 7 = 0$ (2)

$4t - 4 = 4s - 2$

$4t - 4s - 2 = 0$ (3)

$-3 - 2s = 4 - 8t$

$8t - 2s - 7 = 0$ (4)

$12t + 4s + 14 = 0$

$4t - 4s - 2 = 0$

$16t + 12 = 0$

$t = \frac{-12}{16}$

$\boxed{t = -3/4}$

THANK YOU