JEE-MAINS - MATHEMATICS

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JEE-MAINS-2021 Anticipated Questions

Rizee
THE PERFECT GUIDE

1. The value of constant $a > 0$ such that $\int_0^a [\tan^{-1} \sqrt{x}]\, dx = \int_0^a [\cos^{-1} \sqrt{x}]\, dx$ is denotes G.I.F.

(A) $\frac{2(3+\cos 4)}{1-\cos 4}$

(B) $\frac{(3-\cos 4)}{1+\cos 4}$

(C) $\frac{2(3+\cos 4)}{1+\cos 4}$

(D) $\frac{(3+\cos 4)}{1-\cos 4}$

Chapter Name : DEFINITE INTEGRALS AND AREA

Topic Name : PROPERTIES OF DEFINITE INTEGRALS (APPLICATION)

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Explanation : (A)

Given, $\int_0^{\tan^2 1} 0\,dx + \int_{\tan^2 1}^{a} 1\,dx = \int_0^{\cot^2 1} 1\,dx + \int_{\cot^2 1}^{a} 0\,dx$

$a - \tan^2 1 = \cot^2 1 \Rightarrow a = \frac{\sin^4 1 + \cos^4 1}{\sin^2 1 + \cos^2 1}$

$= \frac{4\left[1 - 2\sin^2 1 + \cos^2 1\right]}{\sin^2 2}$

$= \frac{4\left[1 - \frac{1}{2}\sin^2 2\right]}{\frac{1-\cos 4}{2}}$

$= \frac{4\left[2 - \frac{1-\cos 4}{2}\right]}{1-\cos 4}$

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2. The value of $(21_{C_1} - 10_{C_1}) + (21_{C_2} - 10_{C_2}) - (21_{C_3} - 10_{C_3}) + (21_{C_4} - 10_{C_4}) + \cdots + (21_{C_{10}} - 10_{C_{10}})$ is

(A) $2^{20} - 2^{10}$

(B) $2^{21} - 2^{11}$

(C) $2^{21} - 2^{10}$

(D) $2^{20} - 2^{9}$

Chapter Name : BINOMIAL THEOREM

Topic Name : FINDING REMAINDER, DOUBLE SUMMATION, VANDER WAALS THEOREM, DIVISIBILITY PROBLEMS (APPLICATION)

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Explanation : (A)

$$(21_{C_1} + 21_{C_2} + 21_{C_3} - 21_{C_4} + \cdots + 21_{C_{10}}) - (10_{C_1} + 10_{C_2} + 10_{C_3} + \cdots + 10_{C_{10}})$$

$$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1) = (2^{20} - 2) - (2^{10} - 1)$$

$$= (2^{20} - 2^{10})$$

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3. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then.

Chapter Name : PROBABILITY

Topic Name : CONDITIONAL PROBABILITY AND INDEPENDENT EVENTS

(APPLICATION)

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$

(D) $P(E) = \frac{3}{5}, P(F) = \frac{6}{5}$

Explanation : (A)

Let $P(E) = e$ & $P(F) = f$

$P(e \cup f) - P(e \cap f) = \frac{11}{25}$

$\Rightarrow e + f - 2ef = \frac{11}{25} \quad \ldots (i)$

$P(\bar{e} \cap \bar{f}) = \frac{2}{25}$

$\Rightarrow (1 - e)(1 - f) = \frac{2}{25}$

$\Rightarrow 1 - e - f + ef = \frac{2}{25} \quad \ldots (ii)$

From (i) and (ii)

$ef = \frac{12}{25}$ and $e + f = \frac{7}{5}$

Solving we get , $e = \frac{4}{5}, f = \frac{3}{5}$ Or $e = \frac{3}{5}, f = \frac{4}{5}$

4. If $\log(a+c), \log(a+b), \log(b+c)$ are in A.P. and A, B, C are in H.P. then the value of $a+b$ is given (a, b, c > 0)

(A) 2c

(B) 3c

(C) 4c

(D) 6c

Chapter Name : SEQUENCES AND SERIES,QUADRATIC EQUATIONS

Topic Name : ARITHMETIC PROGRESSION (A.P.),FINDING ROOTS , NATURE OF ROOTS , FORMATION OF QUADRATIC EQUATION,HARMONIC PROGRESSION (H.P.) & ARITHMETICO-GEOMETRIC PROGRESSION (A.G.P.) (APPLICATION, LINKAGE)

Explanation : (A)

$\log(a + c) + \log(b + c) = 2\log(a + b)$

$(a + c)(b + c) = (a + b)^2$

$\Rightarrow ab + c(a + b) + c^2 = (a + b)^2$

also, $c = \frac{2ab}{a+b} \Rightarrow 2ab = c(a + b)$

$\Rightarrow 2ab + 2c(a + b) + 2c^2 = 2(a + b)^2$ ……(2)

From (1) and (2),

$c(a + b) + 2c(a + b) + 2c^2 = 2(a + b)^2$

$2(a + b)^2 - 3c(a + b) - 2c^2 = 0$

$\therefore a + b = \frac{3c \pm\sqrt{9C^2+16C^2}}{4} = \frac{3c\pm5c}{4} = 2c$ or $-\frac{c}{2}$

$\therefore a + b = 2c \quad (\because a,b,c > 0)$

5. Matrix A such that A^2= 2A –I, where I is the identity matrix, Then for n≥2, A^n is equal to

(A) $2^{n-1}A-(n-1)I$

(B) $2^{n-1}A-I$

(C) $nA-(n-1)I$

(D) $nA-I$

Chapter Name : MATRICES

Topic Name : TYPES AND ALGEBRA OF MATRICES,TRANSPOSE AND SPECIAL TYPES OF MATRICES

(APPLICATION)

Explanation : (C)

Given, $A^2 = 2A - I$

Now, $A^3 = A(A^2)$

$= A(2A - I)$

$= 2A^2 - A$

$= 2(2A - I) - A = 3A - 2I$

$A^4 = A(A^3)$

$= A(3A - 2I)$

$= 3A^2 - 2A$

$= 3(2A - I) - 2A = 4A - 3I$

Following this, we can say $A^n = nA - (n - I)I$.

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6. The position vectors of the vertices A,B,C of a triangle are $\vec{i} - \vec{j} - 3\vec{k}$, $2\vec{i} + \vec{j} - 2\vec{k}$ and $-5\vec{i} + 2\vec{j} - 6\vec{k}$ respectively. The length of the bisector AD of the angle BAC where D is on the line segment BC, is

Chapter Name : VECTOR ALGEBRA

Topic Name : ADDITION, SUBTRACTION, SCALAR MULITIPLICATION, POSITION VECTORS, SECTION FORMULA, ANGULAR-BISECTORS. (APPLICATION)

(A) $\frac{15}{2}$

(B) $\frac{1}{4}$

(C) $\frac{11}{2}$

(D) None of these

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Explanation : (A)

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2\vec{i} + \vec{j} - 2\vec{k}) - (\vec{i} - \vec{j} - 3\vec{k}) = \vec{i} + 2\vec{j} + \vec{k}.$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (-5\vec{i} + \vec{j} - 6\vec{k}) - (\vec{i} - \vec{j} - 3\vec{k}) = -6\vec{i} + 3\vec{j} + 3\vec{k}.$$

A vector along the bisector of the angle BAC

$$= \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} + \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\vec{i} + 2\vec{j} + \vec{k}}{\sqrt{1^2 + 2^2 + 1^2}} + \frac{-6\vec{i} + 3\vec{j} - 3\vec{k}}{\sqrt{(-6)^2 + 3^2 + (-3)^2}}$$

$$= \frac{1}{\sqrt{6}}(\vec{i} + 2\vec{j} + \vec{k}) + \frac{1}{3\sqrt{6}}(-6\vec{i} + 3\vec{j} - 3\vec{k}) = \frac{1}{3\sqrt{6}}(-3\vec{i} + 9\vec{j}) = \frac{-\vec{i} + 3\vec{j}}{\sqrt{6}}$$

∴ The unit vector along AD $= \frac{-\vec{i} + 3\vec{j}}{\sqrt{10}}$.

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$\therefore \overrightarrow{AD} = \frac{-\overrightarrow{i} + 3\overrightarrow{j}}{10} AD.$

As D is on BC, $\overrightarrow{BD} = t\overrightarrow{BC}$.

$\therefore \overrightarrow{BA} + \overrightarrow{AD} = t(\overrightarrow{BA} + \overrightarrow{AC})$

Or $-\overrightarrow{i} - 2\overrightarrow{j} - \overrightarrow{k} + \frac{-\overrightarrow{i}+3\overrightarrow{j}}{10} AD = t\{-\overrightarrow{i} - 2\overrightarrow{j} - \overrightarrow{k} - 6\overrightarrow{i} + 3\overrightarrow{j} - 3\overrightarrow{k}\} = t(-7\overrightarrow{i} + \overrightarrow{j} - 4\overrightarrow{k})$

$\Rightarrow -1 - \frac{AD}{10} = -7t, -2 + \frac{3}{10} AD = t, -1 = -4t.$

$\therefore t = \frac{1}{4}$

$\therefore -1 - \frac{AD}{10} = -\frac{7}{4}$ or $\frac{AD}{10} = \frac{3}{4}$ $\therefore AD = \frac{15}{2}$.

7. The coordinates of the foot of the perpendicular drawn from the origin to the line joining the points (-9,4,5) and (10,1,-1) will be

(A) (-3,2,1)

(B) (1,2,2)

(C) (4,5,3)

(D) None of these

Chapter Name : THREE DIMENSIONAL GEOMETRY

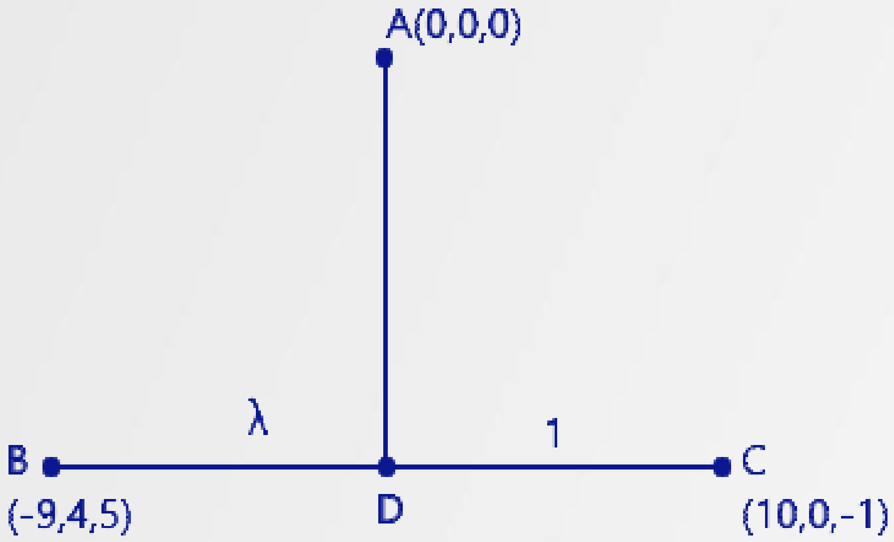
Topic Name : STRAIGHT LINE IN SPACE

(APPLICATION)

Explanation : **(D)**

Let AD be the perpendicular and D be the foot of the perpendicular which

Divides BC in the ratio λ : 1, then $D\left(\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}, \frac{-\lambda+5}{\lambda+1}\right)$



The direction ratios of AD are $\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}$ and $\frac{-\lambda+5}{\lambda+1}$ and direction rations of BC

Are and 19,-4 and -6

Since AD ⊥ BC, we get

$$19\left(\frac{10\lambda-9}{\lambda+1}\right)-4\left(\frac{4}{\lambda+1}\right)-6\left(\frac{-\lambda+5}{\lambda+1}\right)=0$$

$\Rightarrow \lambda = \frac{31}{28}$

Hence, on putting the value of λ in (i),

we get required foot of the perpendicular,

i.e., $\left(\frac{58}{59}, \frac{112}{59}, \frac{109}{59}\right)$

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8. The solution of differential equation $(1+y^2)+\left(x+e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

(A) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c$

(B) $2xe^{\tan^{-1}y}=e^{\tan^{-1}y}+c$

(C) $xe^{\tan^{-1}y}=e^{\tan^{-1}y}+c$

(D) $xe^{\tan^{-1}y}=e^{\tan^{-1}x}+c$

Chapter Name : DIFFERENTIAL EQUATIONS

Topic Name : LINEAR , BERNOULLIOUS EQUATIONS , ORTHOGONAL TRAJECTORIES

(APPLICATION)

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Explanation : (A)

Given equation can be rewritten as

$$\frac{dx}{dy} + \frac{1}{(1+y^2)}x = \frac{e^{\tan^{-1}y}}{(1+y^2)}$$

So IF = $e^{\int \frac{dy}{(1+y^2)}} = e^{\tan^{-1}y}$

∴ Required solution is : $xe^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}\, e^{\tan^{-1}y}}{1+y^2} dy$

Put $e^{\tan^{-1}y} = t \Rightarrow e^{\tan^{-1}y}\frac{dy}{(1+y^2)} = dt$

$\therefore xe^{\tan^{-1}y} = \int t\,dt = \frac{t^2}{2} + c$

$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$

9. In any Δ ABC, $b^2 \sin 2C + c^2 \sin 2B$

(A) Δ

(B) 2Δ

(C) 3Δ

(D) 4Δ

Chapter Name : TRIGONOMETRIC FUNCTIONS

Topic Name : TRIGONOMETRIC RATIOS,MULTIPLES AND SUB-MULTIPLE ANGLES,PROPERTIES OF TRIANGLE

(APPLICATION)

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Explanation : (D)

We have,

$b^2 \sin 2C + c^2 \sin 2B$

$= b^2.(2\sin C) + c^2.(2\sin B\cos B)$

$= 2(b\sin C)(b\cos C) + 2(c\sin B)(c\cos B)$

$= 2(c\sin B)(b\cos c) + 2(c\sin B)(c\cos B)$

$\left[\because \frac{b}{\sin B} = \frac{c}{\sin C}\right]$

$= 2\ c\sin B\ (b\cos C + c\cos B) = 4\Delta$

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10. If the straight line x – 2y + 1 = 0 intersects the circle $x^2 + y^2 = 25$ in points P and Q then the coordinates of the point of intersection tangents drawn at P and Q to the circle $x^2 + y^2 = 25$ are

(A) (25,50)

(B) (-25,-50)

(C) (-25,50)

(D) (25,-50)

Chapter Name : CONIC SECTIONS - CIRCLE

Topic Name : CHORD OF A CONTACT, CHORD BISECT AT A GIVEN POINT LENGTH OF CHORD,POLE - POLAR , CONJUGATE POINTS, LINES, INVERSE POINT

(APPLICATION)

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Explanation : (C)

Let R(h,k) be the point of intersection of tangents drawn at P and Q to the given circle.

Then, PQ is the chord of contact of tangents drawn from R to $x^2 + y^2 = 25$.

So its equation is hx + ky – 25 = 0 …..(i)

It is given that the equation of PQ is

x – 2y + 1 = 0 …..(ii)

Since (i) and (ii) represent the same line

$\therefore \frac{h}{1} = \frac{k}{-2} = -\frac{25}{1}$

⇒ h = - 25,k = 50

Hence, the required point is (-25,50)

11. If p and q are two propositions, then $\sim(p \leftrightarrow q)$ is

(A) $\sim p \wedge \sim q$

(B) $\sim p \vee \sim q$

(C) $(p \wedge \sim q) \vee (\sim p \wedge q)$

(D) None of these

Chapter Name : MATHEMATICAL REASONING

Topic Name : SPECIAL WORDS/PHRASES,BASIC LOGICAL CONNECTIVES , NEGATION OF COMPOUND STATEMENTS TAUTOLOGIES AND CONTRADICTIONS

(APPLICATION)

Explanation : (C)

We know that

$p \rightarrow q \cong \sim p \vee q$ and $q \rightarrow p \cong \sim q \vee p$

$\therefore p \leftrightarrow q \cong (\sim p \vee q) \wedge (\sim q \vee p)$

$\sim (p \leftrightarrow q) \cong \sim (\sim p \vee q) \vee \sim (\sim q \vee p)$

$\sim (p \leftrightarrow q) \cong (p \wedge \sim q) \vee (q \vee \sim p)$

12. If $Z+\frac{1}{Z}=1$ and $a=Z^{2017}+\frac{1}{Z^{2017}}$ and b is the last digit of the number $2^{2^n}-1$,when the integer n > 1, then value of a^2+b^2 is

(A) 23

(B) 24

(C) 26

(D) 27

Chapter Name : COMPLEX NUMBERS

Topic Name : INTEGRAL POWER OF IOTA, ALGEBRAIC OPERATIONS, CONJUGATE OF A COMPLEX NUMBERS

(APPLICATION)

Explanation : (C)

$\because z+\frac{1}{z}=1 \Rightarrow Z^2-Z+1=0$

$$Z=\frac{-(-1)\pm\sqrt{(1-4)}}{2}$$

$=\omega, -\omega^2$ [ω is cube root of unity]

$Z^{2017}=(-\omega)^{2017}=-\omega$

$Z^{2017}=(-\omega^2)^{2017}=-\omega^2$

$\therefore a=Z^{2017}+\frac{1}{Z^{2017}}$

$-\left(\omega+\frac{1}{\omega}\right)=-(\omega+\omega^2)=1$ and $2^{2^n}=2^{4-2^{n-4}}=16^{2^{n-4}}$ has last digit 6

$\therefore b=6-1=5$

Hence, $a^2+b^2=1^2+5^2=26$

13. The sum of infinite terms of a decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval [-2, 3] and the difference between the first two terms is f'(0). Then the common ratio of the GP is

Chapter Name : SEQUENCES AND SERIES

Topic Name : GEOMETRIC PROGRESSION (G.P.) (APPLICATION)

(A) $-\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{2}{3}$

(D) $-\frac{4}{3}$

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Explanation : (C)

Let the GP be $a, ar, ar^2, \ldots\ldots (0 < r < 1)$. From the equestion,

$$\frac{a}{1-r} = 3^2 + 3.3 - 9$$

$\{\because = f'(x) = 3x^2 + 3 > 0$; so, $f(x)$ is monotonically increasing;

$\therefore f(3)$ is the greatest value in $[-2,3].\}$

Also, $f'(0) = 3$. So, $a - ar = 3$.

Solving, $a = 27(1-r)$ and $a(1-r) = 3$

We get $r = \frac{2}{3}, \frac{4}{3}$. But $r < 1$.

14. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(A) 484

(B) 485

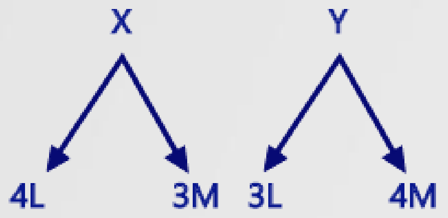
(C) 468

(D) 469

Chapter Name : PERMUTATIONS AND COMBINATIONS

Topic Name : COMBINATIONS WITH & WITHOUT REPETITIONS (CONCEPT)

Explanation : (B)



3	0	0	$3 = 4_{C_3} \times 3_{C_0} \times 3_{C_0} \times 4_{C_3} = 16$
2	1	1	$2 = 4_{C_2} \times 3_{C_1} \times 3_{C_1} \times 4_{C_2} = 324$
1	2	2	$1 = 4_{C_1} \times 3_{C_2} \times 3_{C_2} \times 4_{C_1} = 144$
0	3	3	$0 = 4_{C_0} \times 3_{C_3} \times 3_{C_3} \times 4_{C_0} = 1$

Rizee THE PERFECT GUIDE

15. If $2f(\sin x) + \sqrt{2}\, f(-\cos x) = -\tan x$, then the value of $f\left(\frac{1}{2}\right)$ is $\left(x \in \left(\frac{\pi}{2}, 2\pi\right)\right)$

(A) $\frac{\sqrt{2}-3}{\sqrt{6}}$

(B) $\frac{\sqrt{2}+3}{\sqrt{6}}$

(C) $\frac{\sqrt{2}-3}{\sqrt{5}}$

(D) $\frac{\sqrt{2}+3}{\sqrt{5}}$

Chapter Name : RELATIONS AND FUNCTIONS,TRIGONOMETRIC FUNCTIONS

Topic Name : FUNCTION, DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION,TRANSFORMATION FORMULA AND IDENTITIES (CONCEPT, LINKAGE)

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Explanation : (A)

Put $x = \frac{5\pi}{6}$ and $\frac{2\pi}{3}$

$$2f\left(\frac{1}{2}\right) + \sqrt{2}f\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{3}} \ldots\ldots\ldots (i)$$

$$2f\left(\frac{\sqrt{3}}{2}\right) + \sqrt{2}f\left(\frac{1}{2}\right) = \sqrt{3} \ldots\ldots (ii)$$

by equation $(ii)(2) - \sqrt{2}(i)$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{2} - 3}{\sqrt{6}}$$

16. The mean of a certain number of observations is m. If each observation is divided by $x(\neq 0)$ and increased by y, then mean of the new observations is

(A) $mx + y$

(B) $mx + \frac{y}{x}$

(C) $\frac{m+xy}{x}$

(D) $m + xy$

Chapter Name : STATISTICS

Topic Name : MEAN, MEDIAN AND MODE

(CONCEPT)

Explanation : (C)

Mean (m) = $\frac{x_1+x_2+,,,+x_n}{n}$

New mean = $\frac{\left(\frac{x_1}{x}+y\right)+\left(\frac{x_2}{x}+y\right)+\cdots+\left(\frac{n}{x}+y\right)}{n}$

$= \frac{m}{x} + y = \frac{m+xy}{x}$

17. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq units. Then, the orthocenter of this triangle is at the point

(A) $\left(2, \frac{1}{2}\right)$

(B) $\left(2, -\frac{1}{2}\right)$

(C) $\left(1, \frac{3}{4}\right)$

(D) $\left(1, -\frac{3}{4}\right)$

Chapter Name : STRAIGHT LINES

Topic Name : AREA OF TRIANGLE ,QUADILATERAL, COLLINEARITY , COORDINATES OF PARTICULAR POINTS

(APPLICATION)

Explanation : (A)

Denote the points are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) from the matrix $P = \begin{bmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_3 & y_1 - y_1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 7 & 0 \end{bmatrix}$

$\therefore\ \lambda = \frac{\overrightarrow{R_1}}{|P|} \cdot \frac{\overrightarrow{R_2}}{|P|} = \frac{28}{56} = \frac{1}{2}$

$\therefore$ Circumcentre of triangle is $\left(\frac{7+\frac{1}{2}\times-8}{2} . \frac{-4\times\frac{1}{2}\times-3}{2}\right)$ or $\left(\frac{3}{2}, -\frac{5}{4}\right)$ and centroid is $\left(\frac{5}{3}, -\frac{2}{3}\right)$

then, orthocenter = $\left(\frac{5}{3} \times 3 - 2 \times \frac{3}{2}, -\frac{2}{3} \times 3 + 2 \times \frac{5}{4}\right)$ Or $\left(2, \frac{1}{2}\right)$

18. If $f(x+y) = 2\,f(x)f(y), f'(5) = 1024\,(\log 2)$ and $f(2) = 8$ then the value of $f'(3)$ is

(A) $64\,(\log 2)$

(B) $128\,(\log 2)$

(C) 256

(D) $256\,(\log 2)$

Chapter Name : DERIVATIVES

Topic Name : DERIVATIVE USING FUNCTIONAL EQUATION (APPLICATION)

Explanation : (A)

$$f'(5) = \log_{h\to 0}\frac{f(5+h)-f(5)}{h} = \log_{h\to 0}\frac{2f(5)f(h)-f(5)}{h}$$

$$= \log_{h\to 0}\ 2\,f(5)\left[\frac{f(h)-\frac{1}{2}}{h}\right]$$

$$\Rightarrow 1024\log 2 = 2\,f(5)f'(0)$$

Again now, $f(2+3) = 2\,f(2)f(3)$ ... (i)

$$\Rightarrow \frac{1024\log 2}{2\,f'(0)} = 2\times 8\times f(3)$$

$$\Rightarrow f(3) = \frac{32\log 2}{f'(0)} \quad \ldots.(ii)$$

$$\therefore\ f'(3) = \log_{h\to 0}\frac{2f(3+h)-f(3)}{h} = \log_{h\to 0}\frac{2f(3)f(h)-f(3)}{h}$$

$$= \ 2\,f(3)\,f'(0)$$

$$= 2\times\frac{32\log 2\,f'(0)}{f'(0)} = 64\log 2$$ [from eq (ii)]

19. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(A) $\frac{3\sqrt{2}}{8}$

(B) $\frac{8}{3\sqrt{2}}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\frac{\sqrt{3}}{4}$

Chapter Name : CONIC SECTIONS _ PARABOLA

Topic Name : CHORD OF CONTACT, MID POINT OF CHORD, PAIR OF TANGENT

(APPLICATION)

Explanation : (A)

The shortest distance between y = x - 1 and $y = x^2$ is along the normal of $y^2 = x$.

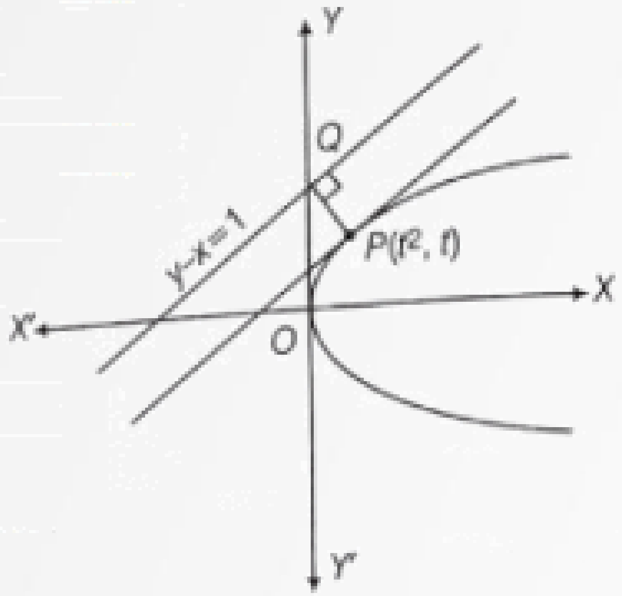
Let $P(t^2, t)$ be any point on $y^2 = x$.

$\therefore$ Tangent at P is $y = \frac{x}{2t} + \frac{t}{2}$

$\therefore$ Slope of tangent $= \frac{1}{2t}$ and tangent at P is parallel to y - x = 1

$\therefore \frac{1}{2t} = 1 \Rightarrow t = \frac{1}{2} \Rightarrow P\left(\frac{1}{4}, \frac{1}{2}\right)$

Hence, shortest distance = PQ $= \frac{\left|\frac{1}{2} - \frac{1}{4} - 1\right|}{\sqrt{1+1}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$



20. $\lim_{x\to\infty} x\left\{\tan^{-1}\frac{x+1}{x+2}-\tan^{-1}\frac{x}{x+2}\right\}$

(A) 1

(B) -1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Chapter Name : INVERSE TRIGONOMETRIC FUNCTIONS,LIMITS

Topic Name : STANDARD LIMTS , ALGEBRA, TRIGNOMETRIC LIMITS, EXPONENTIAL , LOGARITHEMIC LIMITS,INVERSE TRIGONOMETRIC FUNCTIONS USING STANDARD FORMULAS (APPLICATION, LINKAGE)

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Explanation : (C)

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1-xy}$$

$$\lim_{x\to\infty} \left(\frac{\tan^{-1} \frac{x+2}{2x^2+5x+4}}{\frac{x+2}{2x^2+5x+4}} \right) \left(\frac{x+2}{2x^2+5x+4} \right) x$$

21. Let $f(x) = (x - a)(x - b)(x - c)(x - d); a < b < c < d$. Then minimum number of roots of the equation $f''(x) = 0$ is

Answer : 2

Chapter Name : MAXIMA AND MINIMA

Topic Name : LOCAL MIN MAX VALUES (APPLICATION)

Explanation :

$f(a) = f(b) = f(c) = f(d) = 0$

$f(x) = 0$ (4 times). Graph of $f(x)$ will intersect 4 times the x – axis.

So there will be minimum three turnings.

And $f'(x) = 0$ minimum (3 times).

So, $f''(x) = 0$ will be minimum (2 times)

22. If e is eccentricity of the hyperbola $(5x-10)^2+(5y+15)^2=(12x-5y+1)^2$ then $\frac{25e}{13}$ is equal to

Answer : 5

Chapter Name : CONIC SECTIONS _ HYPERBOLA

Topic Name : DIFFERENT FORMS OF HYPERBOLA, POSITION OF POINT, POSITION OF THE W.R.T TO HYPERBOLA (CONCEPT)

Explanation :

Equation can be written as $\sqrt{(x-2)^2+(y+3)^2}=\frac{13}{5}\left|\frac{12x-5y+1}{13}\right|$

So, $e=\frac{13}{5}$

$\frac{25e}{13}=5$

23. Tangents are drawn from points on the line $x - y + 2 = 0$ to the ellipse $x^2 + 2y^2 = 2$, then all the chords of contact pass through the point whose distance from $\left(2, \frac{1}{2}\right)$ is

Answer : 3

Chapter Name : CONIC SECTIONS_ELLIPSE

Topic Name : TANGENT & NORMAL,CHORD OF CONTACT, DIAMETERS OF ELLIPSE

(APPLICATION)

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Explanation :

Consider any point $(t, t+2), t \in R$ on the line $x - y + 2 = 0$

The chord of contact of ellipse with respect to this point is $x(t) + 2y(t+2) - 2 = 0$

$\Rightarrow (4y - 2) + t(x + 2y) = 0, y = \frac{1}{2}, x = -1$

Hence, the point is $\left(-1, \frac{1}{2}\right)$, where distance from $\left(2, \frac{1}{2}\right)$ is 3

24. If $f(x), g(x) = e^x - 1$ and $\int (fog)(x)dx = A\,(fog)(x) + B\tan^{-1}[(fog)(x)] + C$ then $A + B =$

Answer : 0

Chapter Name : INDEFINITE INTEGRALS
Topic Name : METHODS OF INTEGRATION (APPLICATION)

Explanation :

$(fog)(x) = \sqrt{e^x - 1}$

$I = \int \sqrt{e^x - 1}\ dx = \int \frac{2t^2}{t^2+1}$ Where $e^x - 1 = t^2$

$I = 2t - 2\tan^{-1} t + c = 2\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1} + C$

$= 2\ fog(x) - 2\tan^{-1} fog\ (x) + c$

$A + B = 2 - 2 = 0$

25. A possible value of 'K' for which the system of equations $2x - 3y + 6z - 5t = 3, y - 4z + t = 1,$ $4x - 5y + 8z - 9t = k$ in x, y, z has infinite number of solutions.

Answer : 7

Chapter Name : MATRICES

Topic Name : RANK OF MATRICES AND SOLUTIONS OF LINEAR SYSTEM OF EQUATIONS

(APPLICATION)

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Explanation :

$$[AD] = \begin{bmatrix} 2 & -3 & 6 & 3+5t \\ 0 & 1 & -4 & 1-t \\ 4 & -5 & 8 & k+9t \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_1} \begin{bmatrix} 2 & -3 & 6 & 3+5t \\ 0 & 1 & -4 & 1-t \\ 0 & 1 & -4 & k-6-t \end{bmatrix}$$

$$\xrightarrow{R_3 - R_2} \begin{bmatrix} 2 & -3 & 6 & 3+5t \\ 0 & 1 & -4 & 1-t \\ 0 & 0 & 0 & k-7 \end{bmatrix}$$

Infinite solutions $\Rightarrow \text{Rank}(A) = \text{Rank}(AD) \neq 3$

$\Rightarrow K = 7$

26. There is a point (p, q) on the graph of $f(x) = x^2$ and a point (r, s) on the graph of $g(x) = \frac{-8}{x}$, where $p > 0$ and $r > 0$. If the line through (p, q) and (r, s) is also tangent to both the curves at these points, respectively, then the value of $p + q$ is

Answer : 5

Chapter Name : TANGENT & NORMAL

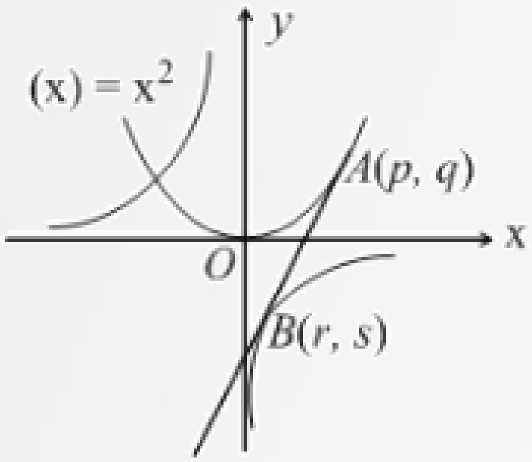
Topic Name : EQUATION OF TANGENT ,NORMAL (APPLICATION)

Explanation :

$y = x^2$ and $y = -\frac{8}{x}; q = p^2$ and $s = -\frac{8}{r}$ (1)

Equating $\frac{dy}{dx}$ at A and B, we get $2p = \frac{8}{r^2}$

$\Rightarrow pr^2 = 4$ (1)



Now, $m_{AB} = \frac{q-s}{p-r} \Rightarrow 2p = \frac{p^2+\frac{8}{r}}{p-r}$

$\Rightarrow p^2 = 2pr + \frac{8}{r} \Rightarrow p^2 = \frac{16}{r}$

$\Rightarrow \frac{16}{r^4} = \frac{16}{r} \Rightarrow r = 1\ (r \neq 0) \Rightarrow p = 4$

$\therefore r = 1, p = 1$

Hence $p + r = 5$

27. The value of the integral $\int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}} \frac{\cos x+\sin x}{1+e^{x-\frac{\pi}{4}}} dx$ is

Answer : 0

Chapter Name : DEFINITE INTEGRALS AND AREA

Topic Name : PROPERTIES OF DEFINITE INTEGRALS (APPLICATION)

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Explanation :

Use $I = \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$2I = \int_{\frac{3x}{4}}^{\frac{5x}{4}} (\sin x + \cos x)dx = 0$$

28. $f(x) = \frac{x}{1+(\ln x)(\ln x)\ldots\infty}\ \forall x \in [1,3]$ is non-differentiable at $x = k$. Then the value of $[k^2]$ is (where $[.]$ represents greatest integer function)

Answer : 7

Chapter Name : CONTINUITY AND DIFFERENTIABILITY

Topic Name : CONTINUITY,DIFFERENTIABILITY (APPLICATION)

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Explanation :

Let $g(x) = (\ln x)\,(\ln x) \ldots \infty$

$$g(x) = \begin{cases} 0, & 1 < x < e \\ 1, & x = e \\ \infty, & e < x < 3 \end{cases}$$

Therefore $f(x) = \begin{cases} x, & 1 < x < e \\ x/2, & x = e \\ 0, & e < x < 3 \end{cases}$

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29. $\frac{1}{x} = \frac{2e}{3!} + \frac{4e}{5!} + \frac{6e}{7!} + \cdots \infty$, then $\int_0^x f(y)\ \log_y x\, dy, \quad y > 1$

Answer : 0

Chapter Name : SEQUENCES AND SERIES

Topic Name : SUM TO N TERMS OF SPECIAL SERIES

(APPLICATION)

Explanation :

$$\frac{1}{x} = 2e\left(\frac{1}{3!} + \frac{2}{5!} + \frac{3}{7!} + \cdots \infty\right)$$

$$= 2e\sum_{r=1}^{\infty} \frac{r}{(2r+1)!} = e\sum_{r=1}^{\infty} \frac{(2r+1)-1}{(2r+1)!}$$

$$= e\sum_{r=1}^{\infty} \left(\frac{1}{(2r)!} - \frac{1}{(2r+1)!}\right)$$

$$= \frac{1}{x} = e\ \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots\right) = \frac{1}{x} = e.(e^{-1}) = x = 1$$

$$\int_0^1 f(y).\log_y dy\ ;\ y > 1 = 0$$

30. The sides of triangle ABC satisfy the relations $a + b - c = 2$ and $2ab - c^2 = 4$, then square of the area of triangle is ___________

Answer : 3

Chapter Name : TRIGONOMETRIC FUNCTIONS
Topic Name : PROPERTIES OF TRIANGLE (APPLICATION)

Explanation :

$a + b - c = 2$

and $2ab - c^2 = 4$ (2)

$\Longrightarrow a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = 4$

$= 2ab - c^2$

$\Longrightarrow (b - c)^2 + (a - c)^2 = 0$

$\Longrightarrow a = b = c$

Triangle is equilateral

$\Longrightarrow a = 2$

$\Longrightarrow \Delta = \sqrt{3}$