TOP 30 ANTICIPATED QUESTIONS-MARCH (MATHEMATICS)

Rizee
THE PERFECT GUIDE

BASED ON JEE-MAINS 2021 ANALYSIS (FEB ATTEMPT)

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1. The contra positive of $(\sim p \wedge q) \rightarrow r$ is

(A) $(p \wedge q) \rightarrow r$

(B) $(p \vee q) \rightarrow r$

(C) $r \rightarrow (p \vee \sim q)$

(D) None of these

Answer : (C)

Explanation :

Contrapositive of $(\sim p \wedge q) \rightarrow r$

$\sim[\sim r \rightarrow (\sim p \wedge q)]$

$\sim(\sim r) \rightarrow (\sim p \wedge q)$

$r \rightarrow p \vee \sim q$

2. The locus point of intersection of tangents to the parabola $y^2 = 4ax$, the angle between them being always 45^0 is

(A) $x^2 - y^2 + 6ax - a^2 = 0$

(B) $x^2 - y^2 - 6ax + a^2 = 0$

(C) $x^2 - y^2 + 6ax + a^2 = 0$

(D) $x^2 - y^2 - 6ax - a^2 = 0$

Answer : (C)

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Explanation :

Equation of tangent is $y = mx + \frac{a}{m}$

$m^2x - my + a = 0$

$\Rightarrow m_1 + m_2 = \frac{y}{x}, m_1m_2 = \frac{a}{x}$

$\tan 45^0 = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \Rightarrow \left(\frac{y}{x}\right)^2 - 4\left(\frac{a}{x}\right) = \left(1 + \frac{a}{x}\right)^2$

$x^2 - y^2 + 6ax + a^2 = 0$

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3. If the function $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x = 0$ then $f(0) =$

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{1}{3}$

Answer : (B)

Explanation :

Applying L-Hospital rule

$$f(0) = \lim_{x\to 0} \frac{e^{x^2}.2x+\sin x}{2x} = \frac{3}{2}$$

4. The domain of the function $f(x) = \sqrt{1 - \sqrt{1 - \sqrt{1 - x^2}}}$ is

(A) $\{x|x < 1\}$

(B) $\{x|x > -1\}$

(C) $[0, 1]$

(D) $[-1, 1]$

Answer : (D)

Explanation :

Clearly $1-x^2 \geq 0, 1-\sqrt{1-x^2} \geq 0, 1-\sqrt{1-\sqrt{1-x^2}} \geq 0.$

$1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1.$

For these values the other two hold.

5. The greatest value of $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$ on $[0, 1]$ is

(A) 1

(B) 2

(C) 3

(D) $\frac{1}{3}$

Answer : (B)

Explanation :

We have $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$

$\therefore\ f'(x) = \frac{1}{3}(x + 1)^{\frac{-2}{3}} - \frac{1}{3}(x - 1)^{\frac{-2}{3}} = \frac{(x-1)^{2/3} - (x+1)^{2/3}}{3(x^2-1)^{2/3}}$

Clearly $f'(x)$ does not exist at $x = \pm 1$

Now $f'(x) = 0$

$\Rightarrow\ (x - 1)^{2/3} = (x + 1)^{2/3}$

$\Rightarrow\ (x - 1)^2 = (x + 1)^2 \Rightarrow -2x = 2x \Rightarrow 4x = 0 \Rightarrow x = 0$

Clearly, $f'(x) \neq 0$ for any other values of $x \in [0, 1]$

The value of f(x) at x = 0 is 2

Hence, the greatest value of f(x) = 2.

6. The angle of intersection of the normal at the point $\left(-\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ of the curves $x^2 - y^2 = 8$ and $9x^2 + 25y^2 = 225$ is

(A) 0

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Answer : (B)

Explanation :

$x^2 - y^2 = 8 \Rightarrow \frac{dy}{dx} = \frac{x}{y} \Rightarrow -\frac{1}{\frac{dy}{dx}} = -\frac{y}{x}$

At the point $\left(-\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), -\frac{1}{\frac{dy}{dx}} = \frac{-\frac{3}{\sqrt{2}}}{-\frac{5}{\sqrt{2}}} = \frac{3}{5}$

Also, $9x^2 + 25y^2 = 225$

$\Rightarrow 18x + 50y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{9x}{25y} \Rightarrow -\frac{dx}{dy} = \frac{25y}{9x}$

At the point $\left(-\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), -\frac{dx}{dy} = \frac{25\times\frac{3}{\sqrt{2}}}{9\left(-\frac{5}{\sqrt{2}}\right)} = -\frac{15}{9} = -\frac{5}{3}$

Since the product of the slopes = –1. Therefore the normal cut orthogonally, i.e., the required angle is equal to $\frac{\pi}{2}$

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7. P and Q are any two points on the circle $x^2 + y^2 = 4$ such that PQ is a diameter. If α and β are the lengths of perpendicular from P and Q on $x + y = 1$ then the maximum value of $\alpha\beta$ is

(A) $\frac{1}{2}$

(B) $\frac{7}{2}$

(C) 1

(D) 2

Answer : (B)

Explanation :

$P(2\cos\theta, 2\sin\theta), Q(-2\cos\theta, -2\sin\theta)$

$$\alpha\beta = \frac{|2\cos\theta + 2\sin\theta - 1|\,|-2\cos\theta - 2\sin\theta - 1|}{2}$$

$$\frac{\left|4(\cos\theta + \sin\theta)^2 - 1\right|}{2} \leq \frac{7}{2}$$

8. The standard deviation for the scores 1, 2, 3, 4, 5, 6 and 7 is 2. Then, the standard deviation of 12, 23, 34, 45, 56 , 67 and 78 is

(A) 2

(B) 4

(C) 22

(D) 11

Answer : (C)

Explanation :

Here, n = 7, sum = 315

$\therefore$ Mean = $\frac{315}{7}$ = 45

Now, standard deviation

$$= \sqrt{\frac{(12-45)^2+(23-45)^2+(34-45)^2+(45-45)^2+(56-45)^2+(67-45)^2+(78-45)^2}{7}}$$

$$= \sqrt{\frac{2(1089+484+121)}{7}} = \sqrt{\frac{3388}{7}}$$

$\sqrt{484}$ = 22

9. From the top of a tower, the angle of depression of a point on the ground is 60^0 If the distance of this point from the tower is $\frac{1}{\sqrt{3}+1}$ m, then the height of the tower is

(A) $\left(\frac{4\sqrt{3}}{2}\right)$ m

(B) $\frac{(\sqrt{3}+3)}{2}$ m

(C) $\frac{(3-\sqrt{3})}{2}$ m

(D) $\frac{\sqrt{3}}{2}$ m

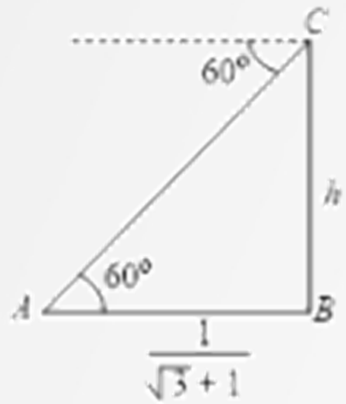
Answer : (C)

Explanation :

Let h be the height of the tower.

$= \frac{h}{l} \Rightarrow h = \frac{\sqrt{3}(\sqrt{3}-1)}{(3-1)}$

$= \frac{3-\sqrt{3}}{2}$ m



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10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times (\vec{b} \times \vec{c})$, then the value of $[\vec{a} \times (\vec{b} \times \vec{c})] \times \vec{c}$ is equal to

(A) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{c}$

(B) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{b}$

(C) $\vec{0}$

(D) $[\vec{a}\,\vec{b}\,\vec{c}]\vec{a}$

Answer : (C)

Explanation :

Given that $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

Again $\vec{a} \times (\vec{b} \times \vec{c}).(\vec{a} \times \vec{c}) = 0$

$\Rightarrow \left[(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}\right].(\vec{a} \times \vec{c}) = 0$

$\Rightarrow (\vec{a}.\vec{c})\ [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\Rightarrow (\vec{a}.\vec{c}) = 0$

$\Rightarrow \vec{a}$ and $\vec{c}$ are perpendicular

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow \left[\vec{a} \times (\vec{b} \times \vec{c})\right] \times \vec{c} = \vec{0}$

11. If α, β be the roots of the equation $x^2 + ax - \frac{1}{2a^2} = 0$, 'a' being a real parameter, then the least value of $[\alpha^4 + \beta^4]$ (where [.] represents greatest integer function)

(A) 1

(B) 2

(C) 3

(D) 4

Answer : (C)

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Explanation :

$\alpha + \beta = -\alpha;\ \alpha\beta = -\frac{1}{2a^2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = a^2 + \frac{1}{a^2}$

$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = a^4 + \frac{1}{2a^4} + 2$

$a^4 + \frac{1}{2a^4} \geq \sqrt{2}$

$\Rightarrow\ \alpha^4 + \beta^4 \geq 2 + \sqrt{2}$

12. The number of terms common between the two series 2 + 5 + 8 + ...up to 50 terms and the series 3 + 5 + 7 + 9 + ...up to 60

(A) 24

(B) 26

(C) 25

(D) None of these

Answer : (D)

Explanation :

Let term of first A.P. be equal to the term of the second A.P. then

2 , 5 , 8 ,....50 terms series 1

3 , 5 ,7 , ..., 60 terms series 2

Common series 5 , 11, 17 , ..., 119

term of series 1 = term of series 2 = 119 = last term of common series

$a = 5, b = 6, a_n = 119$

$a_n = 5 + (n - 1)d$

$\Rightarrow 119 + 1 = 6n$

$\Rightarrow n = 20$

13. The equation of the plane in which the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ lie, is

(A) $17x - 47y - 24z + 172 = 0$

(B) $17x + 47y - 24z + 172 = 0$

(C) $17x + 47y + 24z + 172 = 0$

(D) $17x - 47y + 24z + 172 = 0$

Answer : (A)

Explanation :

The equation of plane, in which the line $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ lies is $a(x-5)+b(y-7)+c(z+3)=0$ ... (i)

Where a, b and c are the direction ratios of the plane. Since, the first line lie on the plane.

∴ Direction ratios of normal to the plane is perpendicular to the direction ratios of line

i.e., $4a+4b-5c=0$ ... (ii)

Since, line $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ lies in this plane. The direction ratios is also perpendicular to this line

∴ $7a+b+3c=0$ ... (iii)

From Eqs. (ii) and (iii), we get

$\frac{a}{17}=\frac{b}{-47}=\frac{c}{24}$

∴ The required equation of plane is $17(x-5)-47(y-7)+(-24)(z+3)=0$

$\Rightarrow 17x-47y-24z+172=0$

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14. The general solution of the differential equation $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$ is

(A) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$

(B) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

(C) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$

(D) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

Answer : (B)

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Explanation :

We have $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$

$\frac{dy}{dx} = \sin\frac{x-y}{2} - \sin\frac{x+y}{2}$

$= -2\cos\frac{x}{2}\sin\frac{y}{2}$

$\Rightarrow \log\tan\frac{y}{4} = -\frac{\sin\frac{x}{2}}{\frac{1}{2}} + c$

$\Rightarrow \log\tan\left(\frac{y}{4}\right) = c - 2\sin\frac{x}{2}$

15. The value of $\left(\frac{^{50}C_0}{1}+\frac{^{50}C_2}{3}+\frac{^{50}C_4}{5}+\cdots+\frac{^{50}C_{50}}{51}\right)$ is

(A) $\frac{2^{50}}{51}$

(B) $\frac{2^{50}-1}{51}$

(C) $\frac{2^{50}-1}{50}$

(D) $\frac{2^{51}-1}{51}$

Answer : (A)

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Explanation :

$$\left(\frac{^{50}C_0}{1}+\frac{^{50}C_2}{3}+\frac{^{50}C_4}{5}+\cdots+\frac{^{50}C_{50}}{51}\right)$$

$$=\frac{1}{1}+\frac{50\times 49}{3\times 2!}+\frac{50\times 49\times 48\times 47}{5\times 4!}+\cdots$$

$$=\frac{1}{51}\left(51+\frac{51\times 50\times 49}{3!}+\frac{51\times 50\times 49\times 48\times 47}{5!}+\cdots\right)$$

$$=\frac{1}{51}\left(51_{C_1}+51_{C_3}+51_{C_5}+\cdots\right)$$

$$=\frac{1}{51}\cdot 2^{51-1}\Rightarrow\frac{2^{50}}{51}$$

16. $\int_0^{2\pi}(\sin x + \lfloor \sin x \rfloor)\, dx$ is equal to

(A) 4

(B) 0

(C) 1

(D) 8

Answer : (A)

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Explanation :

We have, $\int_0^{2\pi}(\sin x + \lfloor \sin x \rfloor)\, dx$

$= \int_0^{\pi}(\sin x + \sin x)\, dx + \int_0^{2\pi}(\sin x - \sin x)\, dx$

$= \int_0^{\pi} 2 \sin x\, dx + 0 = 2\ [-\cos_0^{\pi}]$

$= 2\ (\cos \pi - \cos 0) = 4$

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17. The area bounded by the x-axis, the curve y = f(x) and the lines x = 1, x = b is equal to $\sqrt{b^2+1}-\sqrt{2}$ for all b > 1, then f(x) is

(A) $\sqrt{x-1}$

(B) $\sqrt{x+1}$

(C) $\sqrt{x^2+1}$

(D) $\frac{x}{\sqrt{1+x^2}}$

Answer : (D)

Explanation :

$$\int_1^b f(x)dx = \sqrt{b^2 + 1} - \sqrt{2}$$

$$= \sqrt{b^2 + 1} - \sqrt{1 + 1} = \left[\sqrt{x^2 + 1}\right]_1^b$$

$$\therefore\ f(x) = \frac{d}{dx}\sqrt{x^2 + 1} = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

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18. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then f(A) is equal to

(A) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

Answer : (D)

Explanation :

$$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$f(A) = f^2 + 4x - 5$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

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19. A = $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$, B = (adj A) If A and C = 5A, then $\frac{|\text{adj } B|}{|C|}$ is

(A) 5

(B) 25

(C) -1

(D) 1

Answer : (D)

Explanation :

Since, $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore\ B = \text{adj } A = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

$\Rightarrow\ \text{adj } B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 15 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 625$

$\Rightarrow\ |\text{adj } B| = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 625$

Given that, $C = 5A$

$\Rightarrow\ |C| = 5^3|A| = 125\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 2 \end{bmatrix} = 625$

Hence, $\frac{|\text{adj } B|}{|C|} = \frac{625}{625} = 1$

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20. Let a, b, c are positive real numbers. The following system of equations $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, in x, y and z has

(A) Infinite solutions

(B) Unique solution

(C) No solution

(D) Finite number of solutions

Answer : (B)

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Explanation :

Let $\frac{x^2}{a^2} = X$, $\frac{y^2}{b^2} = Y$ and $\frac{z^2}{c^2} = Z$, then given equation will be

$X + Y – Z = 1$, $X – Y + Z = 1$, $-X + Y + Z = 1$

Here, $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Now, $|A| = -4 \neq 0$

Therefore, the given system of equation has unique solution.

21. If $x^2 + x + 1 = 0$ then the value of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \cdots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$ is

Answer : 54

Explanation :

$x^2 + x + 1 = 0$ Let $x = \omega$

$1 + \omega + \omega^2 = 0$

$\omega^2 = 1$

$$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \left(x^4 + \frac{1}{x^4}\right)^2 + \left(x^5 + \frac{1}{x^5}\right)^2 + \left(x^6 + \frac{1}{x^6}\right)^2 + \cdots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$$

$$\left(\omega + \frac{\omega^2}{\omega^3}\right)^2 + \left(\omega^2 + \frac{\omega}{\omega^3}\right)^2 + \left(\omega^2 + \frac{1}{\omega^3}\right)^2 + \left(\omega + \frac{\omega^2}{\omega}\right)^2 + \left(\omega^2 + \frac{\omega}{\omega^3}\right)^2 + \left((\omega^2)^3 + \frac{\omega}{(\omega^2)^3}\right)^2 + \cdots + \left((\omega^3)^9 + \frac{\omega}{(\omega^3)^9}\right)^2$$

$= -1(-1)^2 + (-1)^2 + (1 + 1)^2 + (-1)^2 + (-1)^2 + (1 + 1)^2 + \cdots + (1 + 1)^2$

$= 9[(-1)^2 + (-1)^2 + (2)^2]$

$= 9(1 + 1 + 4) = 54$

22. The 5 th and 8 th terms of a geometric sequence of real numbers are 7! and 8! respectively. If the sum to first terms of the G.P. is 2205, then n equals

Answer : 3

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Explanation :

Let a, ar, ar^2, ar^3, ... are in G.P.

Now $ar^4 = 7!$ And $ar^7 = 8!$

On dividing, we get $r^3 = 8 \Rightarrow r = 2$

Hence, $a.2^4 = 5040$

$\therefore a = \frac{5040}{16} = 315$

So, 315, 630, 1260, ... are in G.P.

$\therefore S_3 = 2205 \Rightarrow n = 3$

23. Suppose A and Bare two events with $P(A)=0.5$ and $P(A\cup B)=0.8$. let $P(B)=p$ if A and B are mutually exclusive and $P(B)=q$ if A and B are independent events, then the value of q/p is

Answer : 2

Explanation :

When A and B are mutually exclusive, $P(A \cap B) = 0$

$\therefore\ P(A \cup B) = P(A) + P(B)$ (1)

$\Rightarrow 0.8 = 0.5 + p \Rightarrow p = 0.3$ (2)

$P(A \cup B) = P(A) + P(B)$

$= P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

$\Rightarrow 0.8 = 0.5 + q - (0.5)q$

$\Rightarrow 0.3 = \frac{q}{2}$

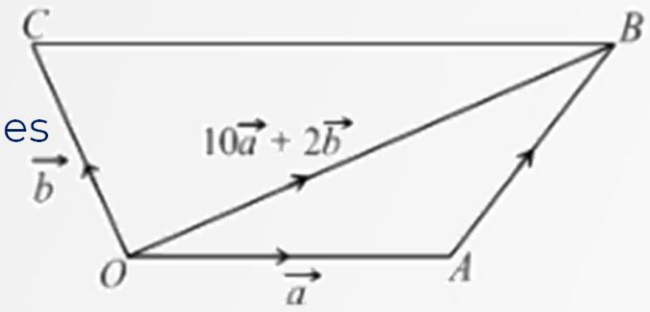
$\Rightarrow q = 0.6$

$\Rightarrow \frac{p}{q} = 2$ (3)

24. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 10\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O,A and C are non-collinear points. Let p denote the area of quadrilateral OACB, and let q denote the area of parallelogram with OA and OC as adjacent sides. If p=k q, then find k

Answer : 6

Explanation :



Here $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 10\,\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$

q = Area of parallelogram with OA and OC as adjacent sides

$\therefore q = |\vec{a} \times \vec{b}|$ ... (i)

p = Area of quadrilateral OABC

= Area of ΔOAB + Area of ΔOBC

$= \frac{1}{2}\left|\vec{a} \times \left(10\,\vec{a} + 2\vec{b}\right)\right| + \frac{1}{2}\left|\left(10\,\vec{a} + 2\vec{b}\right) \times \vec{b}\right|$

$= |\vec{a} \times \vec{b}| + 5|\vec{a} \times \vec{b}|$

$\therefore p = 6|\vec{a} \times \vec{b}|$

Or p = 6q ...[From eq (i)]

$\therefore k = 6$

25. If $f(n+1) = \frac{1}{2}\left\{f(n) + \frac{9}{f(n)}\right\}$ where $n \in N$ and $f(x) > 0\ \forall n \in N$ and $\lim_{n\to\infty} f(n)$ exist then the value of $\lim_{n\to\infty} f(n) =$

Answer : 3

Explanation :

Let $\lim_{n\to\infty} f(n) = 1 \Rightarrow \lim_{n\to\infty} f(n+1) = 1$

$\lim_{n\to\infty} f(n+1) = \frac{1}{2} \lim_{n\to\infty}\left[f(n) + \frac{9}{f(n)}\right]$

$\Rightarrow I = \frac{1}{2}\left[I + \frac{9}{I}\right]$

$2I = \frac{I^2+9}{I} \Rightarrow 2I^2 = I^2 + 9 \Rightarrow I^2 = 9$

$I = 3$

$\because f(n) > 0 \; \forall \; n \in N$

$\therefore \lim_{n\to\infty} f(n) = 3$

26. α and β are the positive acute angles and satisfying equations $5 \sin 2\beta = 3 \sin 2\alpha$ and $\tan \beta = 3 \tan \alpha$ simultaneously. Then the value of $\tan \alpha + \tan \beta$ is ________

Answer : 4

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Explanation :

$5\ \frac{2\tan\beta}{1+\tan^2\beta} = 3\ \frac{2\tan\alpha}{1+\tan^2\alpha}$

$\Rightarrow\ \frac{5\tan\beta}{1+\tan^2\beta} = \frac{3\tan\alpha}{1+\tan^2\alpha}$

Substitute $\tan\beta = 3\tan\alpha$

We have $\frac{5\times 3\tan\alpha}{1+9\tan^2\alpha} = \frac{3\tan\alpha}{1+\tan^2\alpha}$

$\Rightarrow 5 + 5\tan^2\alpha = 1 + 9\tan^2\alpha$

$\Rightarrow 4\tan^2\alpha = 4$

$\Rightarrow \tan\alpha = 1$

i.e., $\tan\beta = 3$

$\therefore \tan\alpha + \tan\beta = 4$

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27. If $\int \frac{dx}{2\sin^2 x + 5\cos^2 x} = \frac{1}{\sqrt{C}} \tan^{-1}\left(\frac{\sqrt{A}\tan x}{\sqrt{B}}\right) + C$ then the value of $\left(\frac{AB}{C}\right)^2$ is ________

Answer : 1

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Explanation :

$\int \frac{dx}{2\sin^2 x + 5\cos^2 x} = \int \frac{\sec^2 x\, dx}{2\tan^2 x + 5}$ ... (1)

[Dividing Numerator and denominator by $\cos^2 x$]

Let $\tan x = t$

$\therefore \sec^2 x\, dx = dt$ (1) becomes

$\therefore \int \frac{dt}{2t^2+5} = \frac{1}{2}\int \frac{dt}{t^2 + \left(\sqrt{\frac{5}{2}}\right)^2} = \frac{1}{2}\frac{\sqrt{2}}{\sqrt{5}}\tan^{-1} + \left(\sqrt{\frac{2}{5}}\, t\right) + C$

$= \frac{1}{\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{5}}\right) + C$

$\therefore A = \sqrt{2}, B = \sqrt{5}, C = \sqrt{10}$

$\left(\frac{AB}{C}\right)^2 = \left(\frac{\sqrt{2}\times\sqrt{5}}{\sqrt{10}}\right)^2 = 1$

28. If N is the number of ways in which a person can walk up a stairway which has 7 steps if he can take 1 or 2 steps up the stairs at a time, then the value of $\frac{N}{3}$ is

Answer : 7

Explanation :

x denotes the number of times he can take unit step and y denotes the number of times he can take 2 steps, then x + 2y = 7

Then we must have x = 1, 3, 5

If x = 1, the steps will be 1 2 2 2

Number of ways = $\frac{4!}{3!}$ = 4

If x = 3, the steps will 1 1 1 2 2

Number of ways = $\frac{5!}{2!\,3!}$ = 10

If x = 5, the steps will 1 1 1 1 1 2

Number of ways = 6_{C_1} = 6

If x = 7, the steps will 1 1 1 1 1 1 1 ⇒ 7_{C_0} = 1

Hence total number of ways = N = 21 ⇒ $\frac{N}{3}$ = 7

29. The number of values of k for which the lines $(k + 1)x + 8y = 4k$ and $kx + (k + 3)y = 3k - 1$ are coincident.

Answer : 1

Explanation :

Lines $(k + 1)x + 8y = 4k$ and $kx + (k + 3)y = 3k – 1$ are coincident then we can compare ratio of coefficients

$\Rightarrow \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$

$\Rightarrow k^2 + 4k + 3 = 8k$ and $24k – 8 = 4k^2 + 12k$

$\Rightarrow (k – 3)(k – 1) = 0$ and $(k – 2)(k – 1) = 0$

$\Rightarrow k = 1$

30. If m is the minimum value of $f(x, y) = x^2 - 4x + y^2 + 6y$ when x and y are subjected to the restrictions $0 \le x \le 1$ and $0 \le y \le 1$, then the value of $|m|$ is

Answer : 3

Explanation :

We have $f(x, y) = x^2 - 4x + y^2 + 6y$

Let $(x, y) = (\cos\theta, \sin\theta)$, then $\theta \in [0, \pi/2]$ and

$f(x, y) = f(\theta) = \cos^2\theta + \sin^2\theta - 4\cos\theta + 6\sin\theta$

$f'(\theta) = 6\cos\theta + 4\sin\theta > 0 \ \forall\ \theta \in [0, \pi/2]$

$\therefore$ $f'(\theta)$ is strictly increasing in $[0, \pi/2]$

$\therefore$ $f(\theta)_{\min} = f(0) = 1 - 4 + 0 = -3$