MATHEMATICS

Section-1

1. For real numbers α and $\beta \neq 0$, if the point of intersection of straight lines χ-α/1 =y-1/2 = z-1/3 and x-4/β = y-6/3 = z-7/3, lies on the plane x+2y-z=8, then α-β is equal to :

A) (1) 5 **B)** (2) 9

C) (3) 3 **D)** (4) 7

Answer: D,
Explanation:

First line is ($\phi+\alpha$, $2\phi+1$, $3\phi+1$)

and second line is ($q\beta+4$, $3q+6$, $3q+7$).

for intersection $\phi+\alpha=q\beta+4$.....(i)

$2\phi+1=3q+6$.....(ii)

$3\phi+1=3q+7$......(iii)

for (ii) &(iii) $\phi=1, q=-1$

so, from (i) $\alpha+\beta=3$

Now, point of intersection is ($\alpha+1$,3,4)

It lies on the plane.

Hence, $\alpha=5$ & $\beta=-2$

2. The point P (a,b) undergoes the following threetransformations successively :

(a)reflection about the line y = x.

(b)translation through 2 units along the positivedirection of x-axis.

(c)rotation through angle $\frac{\pi}{4}$ about the origin inthe anti-clockwise direction.If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ then the value of 2a + b is equal to :

A) 13 **B)** 9

C) 5 **D)** 7

Answer: B,

Explanation:

Image of A(a,b) along y = x is B(b,a). Translatingit 2 units it becomes C(b+2a).

Now, applying rotation theorem

$$-\frac{1}{2}+\frac{7}{\sqrt{2}}i=((b+2)+ai)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$\frac{-1}{2}+\frac{7}{\sqrt{2}}i=\left(\frac{b+2}{\sqrt{2}}-\frac{a}{\sqrt{2}}\right)+i\left(\frac{b+2}{\sqrt{2}}+\frac{a}{\sqrt{2}}\right)$$

$\Rightarrow b-a+2=-1$ ......(i)

$and\ b+2+a=7$ ......(ii)

$\Rightarrow a=4; b=1$

$\Rightarrow 2a+b=9$

3. Let f:R $\rightarrow$ R be defined as

f(x+y)+ f(x-y) = 2f(x) f(y), f(1/2) = -1. Then , the value of $\sum_{k=1}^{20}$ 1/ sin(k) sin(k+f(k)) is equal to:

A) (1) cosec2(21) cos(20) cos(2)

B) (2) sec2(1) sec(21) cos(20)

C) (3) cosec2(4) sec2(21) sin(20) sin(2)(1) cosec(21) sin(20)

D) (4) sec2(21) sin(20) sin(2)

Answer: C,
Explanation:

f(x) = cosλ x

...f(1/2) = -1

so, -1 = cosλ /2

$\Rightarrow \lambda = 2\pi$

Thus f(k) = cos 2 πx

Now k is natural number

Thus f(k) = 1

$\sum_{k=1}^{20}$ 1/sin k sin (k+1) = 1/sin 1 $\sum_{k=1}^{20}$ [sin ((k+1) - k)/sin k . sin (k+1)]

= 1/sin 1 $\sum_{k=1}^{20}$ (cot k - cot (k +1)

= cot 1 - cot 21/sin 1 = cosec 1cosec(21) sin 20

4. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is :

A) 0

B) -1

C) 2

D) 1

Answer: D,

Explanation:

$${}_{10}C_8(25^{(x-1)}+7)\times(5^{(x-1)}+1)^{-1}=180$$

$$\Rightarrow \frac{25^{x-1}+7}{5^{(x-1)}+1}=4$$

$$\Rightarrow \frac{t^2+7}{t+1}=4;$$

$$\Rightarrow t=1,\ 3=5^{x-1}$$

$\Rightarrow x-1=0$ **(one of the possible value)**

$$\Rightarrow x=1$$

5. Let cC be the set of all complex numbers .Let S1 = { $z \in C$:|z -2 | $\leq$ 1} and S2 = { $z \in C$: z(1+i) +z(1-i) $\geq$ 4}. Then the maximum value of| z - 5/2|2 for $z \in S1 \cap S2$ is equal to :

A) (1) $3+2\sqrt{2}/4$

B) (2) $5+2\sqrt{2}/4$

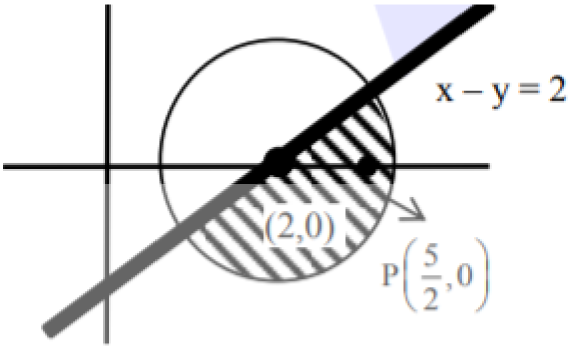
C) (3) $3+2\sqrt{2}/2$

D) (4) $5+2\sqrt{2}/4$

Answer: D,

Explanation:

|t-2| $\leq$ 1 put t = x+iy



(x-2)2 + y2 $\leq$ 1

Also, t(1+i) + t (1-i) $\geq$ 4

Gives x-y $\geq$ 2

Let point on circle be A(2+cos$\pi\theta$,sinθ)

$\theta \in$ [- 3π/4 , π/4]

(AP)2 = (2 + cosθ - 5/2)2 +sin2θ

= cos2θ - cosθ + 1/4 +sin2θ

= 5/4 -= cosθ

For (AP)2 maximumθ = - 3 π/4

(AP)2 = 5/4 +1/$\sqrt{2}$ = 5$\sqrt{2}$ +4 /4$\sqrt{2}$

6. A student appeared in an examination consisting of8 true–false type questions. The student guessesthe answers with equal probability. The smallestvalue of n, so that the probability of guessing atleast 'n' correct answers is less than 1/2 , is:

A) (1) 5 **B)** (2) 6

C) (3) 3 **D)** (4) 4

Answer: A,

Explanation:

P (E) < 1/2

$\Rightarrow \sum_{r=n}^{8}$ **8Cr (1/2)8-r (1/2)r < 1/2**

$\Rightarrow \sum_{r=n}^{8}$ **8Cr (1/2)8 < 1/2**

⇒ 8Cn + 8Cn+1.....+8C8 < 128

⇒ 256 -(8C0+8C1+1....+8Cn-1) < 128

⇒ 8C0 +8C1+.....+8Cn-1 > 128

⇒ n-1 ≥ 4

⇒ n ≥ 5

7. If tan (π/9),x, tan (7π/18) are in arithmeticprogression and tan (π/9),y,tan(5π/18) are also in arithmetic progression,then |x-2y| is equal to:

A) (1) 4 **B)** (2) 3

C) (3) 0 **D)** (4) 1

Answer: C,

Explanation:

x=1/2 (tanπ/9 +tan 7π/18)

and 2y = tanπ/9 + tan 5π/18)-(tanπ/9 +tan 5π/18)

⇒ |x - 2y| = | cot π/9 - tan π/9 /2 - tan 5 π/18|

= |cot 2 π/9 - cot 2 π/9| = 0

(as tan 5 π/18 = cot 2 π/9 ;tan 7 π/18 = cot π/9)

8. Let the mean and variance of the frequency distribution

x: x1=2 x2=6 x3=8 x4=9

f: 4 4 α β

be 6 and 6.8 respectively .If x3 is changed from 8 to 7, then the mean forthe new data will be :

A) (1) 4 **B)** (2) 5

C) (3)17/3 **D)** (4) 16/3

Answer: C,

Explanation:

Given 32+8α +9β = (8 +α +β) x 6

$\Rightarrow$ **2α +3β = 16**

Also, 4x16+4xα +9β = (8 +α +β) x 6.8

$\Rightarrow$ **640 + 40α + 90β = 544 + 68α +68β**

$\Rightarrow$ **28α -22β = 96**

$\Rightarrow$ **14α - 11β = 48 .....(ii)**

from (i) &(iii)

α = 5 &β = 2

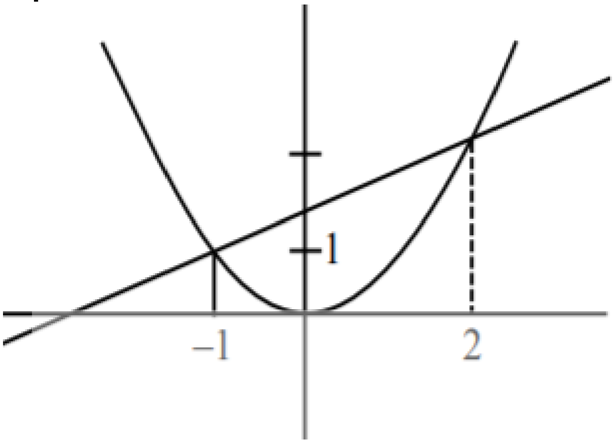
so, new mean = 32+35+18/15 = 85 /15 =17/3

9. The area of the region bounded by y – x = 2 and x2 = y is equal to :-

A) (1) 16/3 **B)** (2) 2/3

C) (3) 9/2 **D)** (4) 4/3

Answer: C,
Explanation:



y - x = 2 ,x2 = y

Now , x2 = 2 + x

⇒ x2 - x - 2 = 0

⇒ (x +1) (x-2) = 0

Area = $\int_{-1}^{2}$ (2+ x - x2)

= |2x +x2 /2 - x3/3|2 -1

= (4 +2 - 8/3) - (-2 +1/2 +1/3)

= 6 - 3 +2 - 1/2 = 9/2

10. 10. Let the y = y(x) be the solution of the differential equation (x - x3)dy = (yx2 - 3x4)dx,x > 2. If y(3) = 3 , then y (4) is equal to :

A) (1) 4 **B)** (2) 12

C) (3) 8 **D)** (4) 16

Answer: B,
Explanation:
(x - x3)dy = (y+yx2 -3x4) dx

⇒ xdy- ydx = (yx2 - 3x4) dx + x3 dy

⇒ xdy - ydx/x2 = (ydx +xdy) - 3x2 dx

d(y/x) = d(xy) - d(x3)

Integrate

y/x = xy - x3 + c

Given f(3) = 3

3/3 = 3x3 -33 +c

c = 19

y/x =xy -x3 +19

at x=4 ,y/4 =4y -64 +19

15y = 4x45

y = 12

Section-2

11. The value of $\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to :

A) 0 **B)** 4

C) -4 **D)** -1

Answer: C,
Explanation:

$$\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$=\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$$

$$\left(\frac{(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})}{\sqrt[4]{1-\sin x}-\sqrt[4]{1+\sin x}}\right)$$

$$\left(\frac{(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})}{\sqrt[2]{1-\sin x}-\sqrt[2]{1+\sin x}}\right)$$

$$=\lim_{x\to 0}\left(\frac{x}{1-\sin x-(1+\sin x)}\right)$$

$$(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})$$

$$(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})$$

$$=\lim_{x\to 0}\frac{x}{(-2\sin x)}(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})$$

$$(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})$$

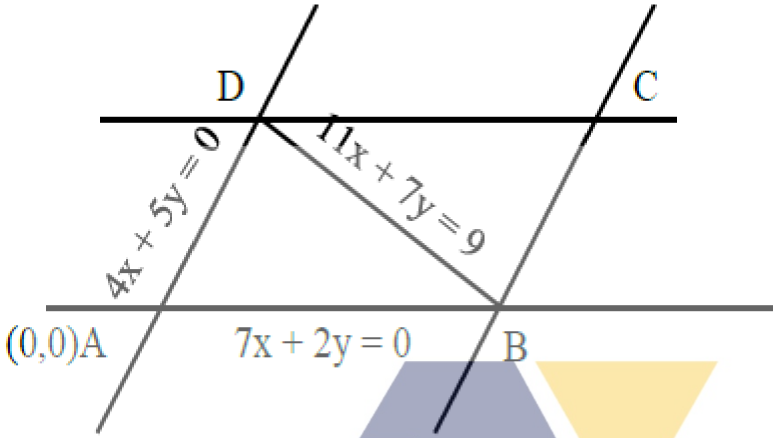
$$=\lim_{x\to 0}\left(-\frac{1}{2}\right)(2)(2)(2)\left\{\because \lim_{x\to 0}\frac{\sin x}{x}=1\right\}=-4$$

12. Two sides of a parallelogram are along the lines 4x + 5y = 0 and 7x + 2y = 0. If the equation of oneof the diagonals of the parallelogram is 11x + 7y = 9, then other diagonal passes through the point:

A) (1,2) **B)** (2,2)

C) (2,1) **D)** (1,3)

Answer: B,
Explanation:
Both the lines pass through origin.



point D is equal of intersection of $4x+5y=0$ & $11x+7y=9$

So, coordinates of point $D=(\frac{5}{3},-\frac{4}{3})$

diagonals of parallelogram intersect at middle let middle point of B,D

$$\Rightarrow(\frac{\frac{5}{3}-\frac{2}{3}}{2},\frac{\frac{-4}{3}+\frac{7}{3}}{2})=(\frac{1}{2},\frac{1}{2})$$

Equation of diagonal AC

$$\Rightarrow(y-0)=\frac{\frac{1}{\alpha}-0}{\frac{1}{\alpha}-0}(\alpha-0)$$

$y=x$

diagonal AC passes through (2, 2).

13. Let $\alpha = \max_{x\in R}\{8^{2\sin 3x}.4^{4\cos 3x}\}$ and $\beta = \min_{x\in R}\{8^{2\sin 3x}.4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of c – b is equal to :

A) 42 **B)** 47

C) 43 **D)** 50

Answer: A,

Explanation:

$\alpha = \max\{8^{2\sin 3x}.4^{4\cos 3x}\}$

$= \max\{2^{6\sin 3x}.2^{8\cos 3x}\}$

$= \max\{2^{6\sin 3x + 8\cos 3x}\}$

$and\ \beta = \min\{8^{2\sin 3x}.4^{4\cos 3x}\} = \min\{2^{6\sin 3x + 8\cos 3x}\}$

$= [-\sqrt{6^2+8^2}, +\sqrt{6^2+8^2}] = [-10, 10]$

$\alpha = 2^{10} \& \beta = 2^{-10}$

$So, \alpha^{1/5} = 2^2 = 4$

$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$

$quadratic\ 8x^2 + bx + c = 0, c - b = 8 \times [(product\ of\ roots] + (sum\ of\ roots)$

$= 8 \times [4 \times \frac{1}{4} + 4 + \frac{1}{4}] = 8 \times [\frac{21}{4}] = 42$

Now range of 6 sin 3x + 8 cos 3x

14. **Let** $f:[0,\infty)\rightarrow[0,3]$ **be a function defined by** $f(x)=\begin{cases}\max\{\sin t:0\le t\le x\}, & 0\le x\le\pi\\ 2+\cos x, & x>\pi\end{cases}$

Then which of the following is true ?

A) f is continuous everywhere but not differentiableexactly at one point in $(0,\infty)$

B) f is differentiable everywhere in $(0,\infty)$

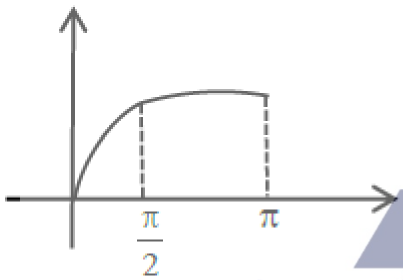
C) f is not continuous exactly at two points in $(0,\infty)$

D) f is continuous everywhere but not differentiable exactly at two points in $(0,\infty)$

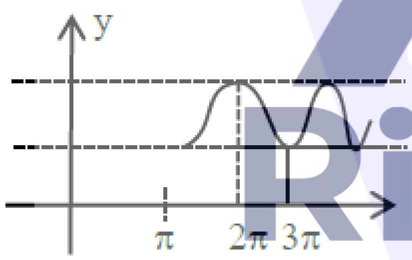
Answer: B,

Explanation:

Graph of max $\{\sin t:0\le t\le x\}$ *in* $x\in[0,\pi]$

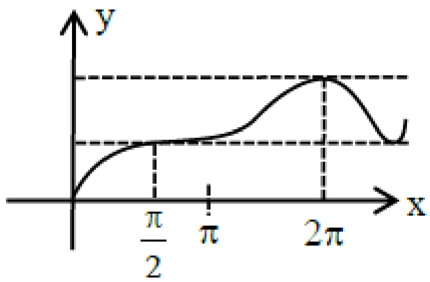


& graph of cos for $x\in[\pi,\infty)$



So graph of

$f(x)=\begin{cases}\max\{\sin t:0\le t\le x, & 0\le x\le\pi\\ 2+\cos x & x>h\end{cases}$



$f(x)$ **is differentiable everywhere in** $(0,\infty)$

15. Let N be the set of natural numbers and a relation R on N be defined by

$R = \{(x,y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$

Then the relation R is :

A) symmetric but neither reflexive nor transitive

B) reflexive but neither symmetric nor transitive

C) reflexive and symmetric, but not transitive

D) an equivalence relation

Answer: B,

Explanation:

$x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow x(x^2 - y^2) - 3y(x^2 - y^2) = 0$

$\Rightarrow (x - 3y)(x - y)(x + y) = 0$

Now $x = y \;\forall\; (x,y) \in N \times N$ **so reflexive**

But not symmetric & transitive

See,(3,1) satisfies but (1,3) does not. Also (3,1) & (1,-1) satisfies but (3,-1) does not

16. Which of the following is the negation of the statement " for all M>0, there exists $x \in S$ such that $x \geq M$ "?

A) there exists M > 0, such that x < M for all $x \in S$

B) there exists M > 0, there exists $x \in S$ such that $x \geq M$

C) there exists M > 0, there exists $x \in S$ such that x < M

D) there exists M > 0, such that $x \geq M$ for all $x \in S$

Answer: A,

Explanation:

P : for all M > 0, there exists $x \in S$ **such that** $x \geq M$ **.**

~ P : there exists M > 0, for all $x \in S$

Such that x < m

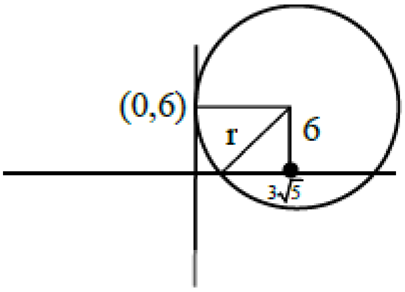
Negation of 'there exsits' is 'for all'.

17. Consider a circle C which touches the y-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the x-axis.Then the radius of the circle C is equal to :

A) $\sqrt{53}$ **B)** 9

C) 8 **D)** $\sqrt{82}$

Answer: B,
Explanation:



$$r=\sqrt{6^2+(3\sqrt{5})^2}$$
$$=\sqrt{36+45}=9$$

18. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}=\vec{b}\times(\vec{b}\times\vec{c})$.If magnitudes of the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are $\sqrt{2}, 1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta(0<\theta<\frac{\pi}{2})$ then the value of 1+ $\tan\theta$ is equal to :

A) $\sqrt{3}+1$ **B)** 2

C) 1 **D)** $\frac{\sqrt{3}+1}{\sqrt{3}}$

Answer: B,
Explanation:

$$\vec{a}=(\vec{b}.\vec{c})\vec{b}-(\vec{b}.\vec{b})\vec{c}$$
$$=1.2\cos\theta\,\vec{b}-\vec{c}$$
$$\Rightarrow \vec{a}=2\cos\theta\,\vec{b}-\vec{c}$$
$$|\vec{a}|^2=(2\cos\theta)^2+2^2-2.2\cos\theta\,\vec{b}.\vec{c}$$
$$\Rightarrow 2=4\cos^2\theta+4-4\cos\theta.2\cos\theta$$
$$\Rightarrow -2=-4\cos^2\theta$$
$$\Rightarrow \cos^2\theta=\frac{1}{2}$$
$$\Rightarrow \sec^2\theta=2$$
$$\Rightarrow \tan^2\theta=1$$
$$\Rightarrow \theta=\frac{\pi}{4}$$
$$1+\tan\theta=2$$

19. Let A and B be two 3×3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to

A) 2 **B)** 4

C) 1 **D)** 0

Answer: D,
Explanation:

$C = A^2 - B^2; |C| \neq 0$

$A^5 = B^5$ and $A^3B^2 = A^2B^2$

$Now, A^5 - A^3B^2 = B^5 - A^2B^3$

$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$

$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$

Post multiplying inverse of $A^2 - B^2$

$A^3 + B^3 = 0$

20. Let $f:(a,b) \to \mathbf{R}$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiablefunction g(x). If $f(x) = 0$ has exactly five distinct roots in (a, b), then g(x)g'(x) = 0 has at least :

A) twelve roots in (a, b) **B)** five roots in (a, b)

C) seven roots in (a, b) **D)** three roots in (a, b)

Answer: C,
Explanation:

$$f(x) = \int_a^x g(t)dt$$

$f(x) \to 5$

$f'(x) \to 4$

$g(x) \to 4$

$g'(x) \to 3$

21. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. if $\vec{a}.\vec{b} = -1$ and $\vec{b}.\vec{c} = 10$, then $(\vec{a} \times \vec{b}).\vec{c}$ is equal to _____.

Answer: ____________________

Answer: 9

Explanation:

$\vec{a} = (1, -\alpha, \beta)$

$\vec{b} = (3, \beta, \alpha)$

$\vec{c} = (-\alpha, -2, 1); \alpha, \beta \in I$

$\vec{a}.\vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1$

$\Rightarrow \alpha\beta = 2$

1 2

2 1

-1 -2

-2 -1

$\vec{b}.\vec{c} = 10$

$\Rightarrow -3\alpha - 2\beta - \alpha = 10$

$\Rightarrow 2\alpha + \beta + 5 = 0$

$\therefore \alpha = -2, \beta = -1$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

$= 1(-1+4) - 2(3-4) - 1(-6+2)$

$= 3 + 2 + 4 = 9$

22. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points. Q(3, –4, –5) and R(2, –3, 1) and the plane 2x + y + z = 7, is equal to______.

Answer: ____________________

Answer: 7

Explanation:

$\overrightarrow{QR} = \frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$

$\Rightarrow (x, y, z) \equiv (r+3, -r-4, -6r-5)$

Now, satisfying it in the given plane.

We get r = –2.

so, required point of intersection is T(1,–2,7).

Hence, PT = 7.

23. If the real part of the complex number $z = \frac{3+2i\cos}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta =$

Answer: ____________________

Answer: 1

Explanation:

$$Re(z) = \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, $\sin^2 3\theta + \cos^2\theta = 1$

24. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, – 4), one focus at (4, – 4) and one vertex at (5, – 4). If mx – y = 4, m > 0 is a tangent to the ellipse E, then the value of $5m^2$ is equal to _______

Answer: ____________________

Answer: 3

Explanation:

Given $C(3, -4), S(4, -4)$

and $A(5, -4)$

hence $a = 2$ & $ae = 1$

$$\Rightarrow e = \frac{1}{2}$$

$$\Rightarrow b^2 = 3$$

So, $E = \frac{(x-3)^2}{4} + \frac{(y+4)^2}{3} = 1$

Intersecting with given tangent.

$$\frac{x^2-6x+9}{4} + \frac{m^2x^2}{3} = 1$$

Now, D = 0 (as it is tangent)

So, $5m^2 = 3$

25. If $\int_0^x (\sin^3 x)\, e^{-\sin^2 x}\, dx = \alpha = \frac{\beta}{e}\int_0^1 \sqrt{t}\, e^t\, dt$, then $\alpha+\beta =$ ______

Answer: ____________________

Answer: 5

Explanation:

$$I = 2\int_0^{\pi/2} \sin^3 x\, e^{-\sin^2 x}\, dx$$

$$= 2\int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx + \int_0^{\pi/2} \cos x\, e^{-\sin^2 x}(-\sin 2x)\, dx$$

$$= \int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx + \left[\cos x\, e^{-\sin^2 x}\right]_0^{\pi/2} + \int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx$$

$$= 3\int_0^{\pi/2} \sin x\, e^{-\sin^2 x}\, dx - 1$$

$$= \frac{3}{2}\int_{-1}^{0} \frac{e^{\alpha}\, d\alpha}{\sqrt{1+\alpha}} - 1 \quad \textbf{(Put } -\sin^2 x = t\textbf{)}$$

$$= \frac{3}{2e}\int_0^1 \frac{e^x}{\sqrt{x}}\, dx - 1 \quad \textbf{(Put } 1+\alpha = x\textbf{)}$$

$$= \frac{3}{2e}\int_0^1 e^x \frac{1}{\sqrt{x}}\, dx - 1$$

$$= 2 - \frac{3}{e}\int_0^1 e^x \sqrt{x}\, dx$$

Hence, $\alpha+\beta = 5$

26. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to ________

Answer: ____________________

Answer: 2

Explanation:

$t^4 - t^3 - 4t^2 - t + 1 = 0,\ e^x = t > 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow \alpha^2 - \alpha - 6 = 0,\ \alpha = t + \frac{1}{t} \ge 2$

$\Rightarrow \alpha = 3,\ -2$ (reject)

$\Rightarrow t + \frac{1}{t} = 3$

The number of real roots = 2

27. Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y} dx; \alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to____.

Answer: ____________________

Answer: 2

Explanation:

$$\int e^{-y} dy = \int e^{\alpha x} dx$$

$$\Rightarrow e^{-y} = \frac{e^{\alpha x}}{\alpha} + c \quad ...(i)$$

put (x, y) = (ln 2, ln 2)

$$\frac{-1}{2} = \frac{2^{\alpha}}{\alpha} + C \quad ...(ii)$$

Put (x, y) = (0, - ln 2) in (i)

$$-2 = \frac{1}{\alpha} + C \quad ...(iii)$$

(ii) - (iii)

$$\frac{2^{\alpha} - 1}{\alpha} = \frac{3}{2}$$

$$\Rightarrow \alpha = 2 \text{ (as } \alpha \in N)$$

28. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10}.(11)^{11}.(13)^{13}$ is equal to_____.

Answer: ____________________

Answer: 924

Explanation:

$$N = 2^{10} \times 5^{10} \times 11^{11} \times 13^{13}$$

Now, power of 2 must be zero,

power of 5 can be anything,

power of 13 can be anything.

But, power of 11 should be even.

So, required number of divisors is $1 \times 11 \times 14 \times 6 = 924$

29. Let $A=\{n \in N | n \le n+10,000\}$, $B=\{3k+1 | k \in N\}$ and $C=\{2k | k \in N\}$, then the sum of all the elements of the set $A \cap (B-C)$ is equal to_______

Answer: _____________________

Answer: 832

Explanation:

$B-C=\{7, 13, 19, ..., 97, ...\}$

Now, $n^2-n \le 100 \le 100$

$\Rightarrow n(n-1) \le 100 \times 100$

$\Rightarrow A=\{1, 2, ..., 100\}$

So, $A \cap (B-C)=\{7, 13, 19, ..., 97\}$

Hence, Sum $=\frac{16}{2}(7+97)=832$

30. If $A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M=A+A^2+A^3+...+A^{20}$, then the sum of all the elements of the matrix M is equal to _____

Answer: _____________________

Answer: 2020

Explanation:

$$A^n=\begin{bmatrix} 1 & n & \frac{n^2+}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$=20\times 3+2\times\left(\frac{20\times 21}{2}\right)+\sum_{r=1}^{20}\left(\frac{r^2+r}{2}\right)$$

So, required sum $=60+420+105+35\times 41=2020$