25-07-2021_SHIFT-2_MATHS_MEMORY BASED

1. If matrix $P=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$, then the matrix P^{50} is equal to

1) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$ 2) $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$ 3) $\begin{bmatrix}1 & 0\\ 75 & 1\end{bmatrix}$ 4) $\begin{bmatrix}1 & 1\\ 25 & 1\end{bmatrix}$

Key:-2

Sol:-

$$P^2=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$$

$$P^3=\begin{bmatrix}1 & 0\\ \frac{2}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$$

$$P^3=\begin{bmatrix}1 & 0\\ \frac{3}{2} & 1\end{bmatrix}$$

Similarly

$$P^{50}=\begin{bmatrix}1 & 0\\ \frac{50}{2} & 1\end{bmatrix}$$

$$P^{50}=\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$$

2. If the first sample A of 100 items has mean 15 and standard deviation 3 and second sample B has 150 items. If the combined mean and standard deviation of items of both the sample is 15.6 and $\sqrt{13.44}$. Then the standard deviation of items if sample B is.

Ans 4

Sol. Combined mean $=15.6$

$$\therefore 15.6=\frac{100\times 15+150\times \overline{x}_B}{250}$$

$$\Rightarrow \overline{x}_B=16 \qquad \text{(mean of sample B)}$$

Combined standard deviation= $\sqrt{13.44}$

$\Rightarrow$ combined variance $\left(\sigma^2\right) = 13.44$

$$\sigma^2 = \frac{\sum x_i^2}{n}(\bar{x})^2$$

$$13.44 == \frac{\sum x^2}{250} - 243.36$$

$$\Rightarrow \sum x_i^2 = 64200 \qquad \ldots\ldots(1)$$

For sample A

$$9 = \frac{\sum x_{iA}^2}{100} - 225$$

$$\Rightarrow \sum x_{iA}^2 = 23400$$

$$\Rightarrow \sum x_{iB}^2 = 64200 - 23400 = 40800$$

Standard deviation of sample B will be

$$\sqrt{\frac{\sum x_{iB}^2}{n_B} - \left(\bar{x}_B\right)^2} = \sqrt{\frac{40800}{150} - 256} = 4.$$

3. The value of $x \in \left[\frac{\pi}{4}, \frac{\pi}{4}\right]$ for which $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

1) $-\frac{\pi}{4}$ 2) $-\frac{\pi}{8}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{8}$

Ans 3

Sol.

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_2 + R_3$$

$$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \sin x + 2\cos x & \sin x + 2\cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x+2\cos x)\begin{vmatrix}1 & 1 & 1\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix}=0$$

$$C_2\rightarrow C_2-C_1, C_3\rightarrow C_3-C_1$$

$$(\sin x+2\cos x)\begin{vmatrix}1 & 0 & 0\\ \cos x & \sin x-\cos & 0\\ \cos x & 0 & \sin x-\cos x\end{vmatrix}=0$$

$$(\sin x+2\cos x)(\sin x-\cos x)^2=0$$

$\sin x=\cos x$ *or* $\sin x=-2\cos x$

$\tan x=1$ *or* $\tan x=-2$

$$\therefore x\in\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$$

$$x=\frac{\pi}{4}$$

4. If $\vec{a}$ and $\vec{b}$ are two vectors such that , $|\vec{a}\times\vec{b}|=8, |\vec{a}|=2, |\vec{b}|=5$, then the value of $|\vec{a}.\vec{b}|$ is.

1) 6 2)3 3) 12 4) 9

Ans. 1

Sol. $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

$8=2\times5\times\sin\theta$

$\sin\theta=\frac{4}{5}\Rightarrow\cos\theta=\pm\frac{3}{5}\Rightarrow|\cos\theta|=\frac{3}{5}$

$|\vec{a}.\vec{b}|=|\vec{a}||\vec{b}||\cos\theta|=2\times5\times\frac{3}{5}=6$

5. If function $f(x):A\rightarrow B,$ and $g(x):B\rightarrow C$ are defined such that $(g(f(x)))^{-1}$ exist then $f(x)$ and $g(x)$ are.

1)One-one and onto 2) many-one and onto

3) one-one and into 4) many-one and into

Ans. 1

Sol. Clearly $g\left(f\left(x\right)\right)$ is one-one onto so $f\left(x\right)$ and $g\left(x\right)$ both are one- one and

6. If $a+b+c=1, ab+bc+ca=2$ and $abc=3,$ then the value of $a^4+b^4+c^4$ is

Ans. 13

Sol. $(a+b+c)^2=1$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=1$

$\Rightarrow a^2+b^2+c^2=-3 \qquad ....(i)$

$\Rightarrow ab+bc+ca=2 \qquad (ii)$

Squaring of equation (ii)

$\Rightarrow a^2b^2+b^2c^2+c^2a^2+2\left(ab^2c+bc^2a+ca^2b\right)=4$

$\Rightarrow a^2b^2+b^2c^2+c^2a^2+2abc(a+b+c)=4$

$\Rightarrow a^2b^2+b^2c^2+c^2a^2+6=4$

$\Rightarrow a^2b^2+b^2c^2+c^3a^2=-2 \qquad ...(iii)$

Squaring of equation (i)

$\Rightarrow a^4+b^4+c^4+2\left(a^2b^2+b^2c^2+c^2a^2\right)=9$

$\Rightarrow a^4+b^4+c^4-4=9$

$\Rightarrow a^4+b^4+c^4=13$

7. Which of the following value is just greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$

1) 2 2) 3 3) 4 4) 5

Ans. 2

Sol. $Let 10^{100}=n$

$So, \left(1+\frac{1}{n}\right)^n={}^nC_0+{}^nC_1\left(\frac{1}{n}\right)+{}^nC_2\left(\frac{1}{n}\right)^2+{}^nC_3\left(\frac{1}{n}\right)^3+....$

$=1+1+\frac{n(n-1)}{2n^2}+\frac{n(n-1)(n-2)}{6n^3}+....$

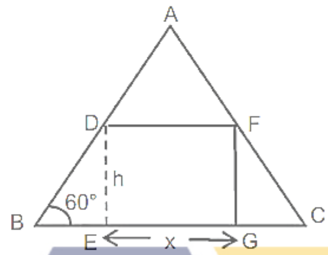
$\Rightarrow \qquad \left(1+\frac{1}{n}\right)^n>2$

Also $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e < 3.$

8. If a rectangle is inscribed in an equilateral triangle of side $2\sqrt{2}$, then the square of maximum area of rectangle is

Ans. 3

Sol.



Area of rectangle $= x.h \quad ....(i)$

From ΔBDE

$h = BE \tan 60$

$$h = \frac{\left(2\sqrt{2}-x\right)}{2}\sqrt{3} \qquad ...(ii)$$

$$\text{So area, A } = \frac{\sqrt{3}}{2}\left(2\sqrt{2}\times - x^2\right)$$

$$\text{For maxima } \frac{dA}{dx} = \frac{\sqrt{3}}{2}\left(2\sqrt{2}-2\times\right) = 0$$

$$\text{From (ii) } h = \sqrt{\frac{3}{2}}$$

$\text{Area} = x.h = \sqrt{3}$

$(\text{Area})^2 = 3$

9. If the coefficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal then the value of n is:

1) 53 2) 54 3) 55 4) 56

Ans. 3

Sol.

$$\left(2+\frac{x}{3}\right)^n = \sum_{r=0}^{n} {}^nC_7\, 2^{n-r}\left(\frac{x}{3}\right)^r$$

Coefficient of $x^7 = {}^nC_7 2^{n-7}\left(\frac{1}{3}\right)^7$

Coefficient of $x^8 = {}^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8$

$$\therefore {}^nC_7\frac{2^{n-7}}{3^7} = {}^nC_8\frac{2^{n-8}}{3^8}$$

$$\Rightarrow {}^nC_7.6 = {}^nC_8$$

$$\Rightarrow \frac{6.n!}{7!.(n-7)!} = \frac{n!}{8!(n-8)!}$$

$$= 48 = n-7 \Rightarrow n = 55$$

10. If $f(x)=\begin{cases}\frac{P(x)}{x-2} & ;x\neq 2\\ 7 & ;x=2\end{cases}$ and $P(x)$ is a polynomial such that $p''(x)$ is constant and $P(3)=9$. If $f(x)$ is continuous at $x=2$ then find the value of $P(5)$

Ans. 39

Sol. $P(x)=K(x-2)(x-\beta)$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{K(x-2)(x-\beta)}{(x-2)}$$

$$\Rightarrow K(2-\beta)=7....(1)$$

And $P(3)=K(3-2)(3-\beta)=9$

$$K(3-\beta)=9...(2)$$

Divide equation (1) by (2)

$$\frac{2-\beta}{3-\beta}=\frac{7}{9}\Rightarrow \beta=\frac{-3}{2}$$

So, K=2

Then $P(x)=2(x-2)\left(\times+\frac{3}{2}\right)$

$$P(5) = 2\times(5-2)\times\left(5+\frac{3}{2}\right) = 39$$

11. The number of real solutions of the equation $x^2 - |x| - 12 = 0$ is.

1) 0 2) 1 3) 3 4) 2

Ans. 4

Sol. $|x|^2 - |x| - 12 = 0$

$|x| = 4, -3$ (not possible)

$\Rightarrow |x| = 4 \Rightarrow x = \pm 4$

$\therefore$ Number of real solutions=2

12. A coin is tossed n times. If the probability of getting at least one head is greater than 0.9, then the minimum value of n is.

1) 3 2) 5 3) 4 4) 2

Ans. 3

Sol. $1-\left(\frac{1}{2}\right)^n > 0.9$

$\Rightarrow 0.1 > \left(\frac{1}{2}\right)^n \Rightarrow n = 4$

13. Negation of the statement:

"we will play football only if ground is not wet and there is no sunlight" is

1) we will play football if ground is wet and there is no sunlight.

2) We will play football if ground is wet and there is sunlight.

3) There is no sunlight and ground is not wet and we will not play football.

4) There is sunlight of ground is wet and we will play football.

Ans. 4

Sol. P: we will play football

q: Ground is not wet

r: There is no sunlight.

$\therefore$ Given statement is $p \to (q < r)$

$\therefore$ Negation is $P \wedge \sim (q \wedge r)$

$P \wedge \sim (q \wedge r)$

14. If ${}^nC_0 + 3\,{}^nC_1 + 5\,{}^nC_2 + 7\,{}^nC_3 + ...$ till $(n+1)$ terms, is equal to $2^{100}.101$, then the value of $2\left[\dfrac{n-1}{2}\right]$

(where [.] represents G.I.F)

1) 98 2)97 3)96 4)100

Ans. 4

Sol. ${}^nC_0 + 3\,{}^nC_1 + 5\,{}^nC_2 + 7\,{}^nC_3 + ...(n+1)$ terms$= \sum\limits_{r=0}^{n} (2r+1)\,{}^nC_r$

$= 2\sum\limits_{r=0}^{n} r\,{}^nC_r + \sum\limits_{r=0}^{n} {}^nC_r$

$= 2n.2^{n-1} + 2^n = (n+1).2^n$

15. Evaluate $\int\limits_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right)dx$

1) 0 2) 1 3)2 4)3

Ans. 1

Sol. $1 = \int\limits_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right)dx$

$f(x) = \log\left(\sqrt{x^2+1+x}\right)$

$f(-x) = \log\left(\sqrt{x^2+1-x}\right)$ $f(-x) = \log\left(\sqrt{x^2+1}-x\right)$

$= -f(x)$

So $f(x)$ is a odd function.

$\Rightarrow I = 0$

16. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of n is

Ans. 3

Sol. ${}^{n}C_r = {}^{n}C_{r-1} \Rightarrow \frac{n-r+1}{r} = 1 \Rightarrow n+1 = 2r....(1)$

And ${}^{n}P_r = {}^{n}P_{r+1} p \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow n-r-1$

Solving $(1) \& (2)$ $n+1 = 2(n-1) \Rightarrow n = 3$

17. Value of $\sum_{n=8}^{100}\left[(-1)^n \frac{n}{2}\right]$:(where [.] represent greatest integer function)

Ans. 4

Sol. $\sum_{n=8}^{100}\left[(-1)^n \frac{n}{2}\right]$

$\Rightarrow \underbrace{[4]+[-4.5]}_{-1}+\underbrace{[5]+[-5.5]}_{-1}+.....[49]+[-49.5]+[50]$

$\Rightarrow -1\times 46 + 50 = 4$

18. Evaluate $\cot\left(\frac{\pi}{24}\right)$

1) $\sqrt{6}-\sqrt{3}+\sqrt{2}-2$

2) $\sqrt{6}+\sqrt{3}-\sqrt{2}+2$

3) $\sqrt{6}+\sqrt{3}+\sqrt{2}-2$

4) $\sqrt{6}+\sqrt{3}+\sqrt{2}+2$

Ans. 4

Sol. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{1+\cos 2\theta}{\sin 2\theta}$

$\therefore \cos\frac{\pi}{24} = \frac{1+\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}\left(\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)\&\left(\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$\Rightarrow \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$

$$\Rightarrow \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}=\sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

19. If two lines $L_1 \equiv \frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are coplanar. Then the value of k is

1) -1 2) 1 3) 2 4) -2

Ans. 2

Sol. $\begin{vmatrix} k+1 & 2+2 & 3+3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix}=0$

$\Rightarrow (k+1)(-4)-4(-8)+6(2-6)=0$

$\Rightarrow (k+1)(-4)=-8$

$k=1$

20. If $y=f(x)$ is the solution of differential equation $xdy=\left(y+x^3\cos x\right)dx\ and\ f(\pi)=0$ then $f\left(\frac{\pi}{2}\right)$ is.

1) $\frac{\pi^2}{4}+\frac{\pi}{6}$ 2) $\frac{\pi^2}{4}+\frac{\pi}{2}$ 3) $\frac{\pi^2}{6}+\frac{\pi}{4}$ 4) $\frac{\pi^2}{6}+\frac{\pi}{6}$

Ans. 2

Sol. $\frac{xdy-ydx}{x^2}=x\cos xdx$

$\Rightarrow \int d\left(\frac{y}{x}\right)=\int x\cos xdx$

$\Rightarrow \frac{y}{x}=x\sin x+\cos x+c$

$\Rightarrow 0=0-1+c \Rightarrow c=1$

$\Rightarrow y=x^2\sin x+x\cos x+x$

$\Rightarrow f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}+0+\frac{\pi}{2}=\frac{\pi^2}{4}+\frac{\pi}{2}$

21. If three vector $i6+\hat{k}, b\hat{i}+n\hat{j}+c\hat{k}\ and\ a\hat{i}+a\hat{j}+b\hat{k}$ are coplanar then

1) $a^2 = bc$ 2) $b^2 = ac$ 3) $c^2 = ab$

Ans. 2

Sol. $\begin{vmatrix} 1 & 0 & 1 \\ b & b & c \\ a & a & b \end{vmatrix} = 0$

$1(b^2 - ac) + 1(ab - ab) = 0 \Rightarrow b^2 = ac$

22. If $\vec{x}$ & $\vec{y}$ are two vectors such that $|\vec{x}| = |\vec{y}|$ & $|\vec{x} - \vec{y}| = n|\vec{x} + \vec{y}|$, , then the angle between $\vec{x}$ & $\vec{y}$

1) $\cos^{-1}\left(\frac{1-n}{1+n}\right)$

2) $\cos^{-1}\left(\frac{n^2+1}{1-n^2}\right)$

3) $\cos^{-1}\left(\frac{n+1}{n-2}\right)$

4) $\cos^{-1}\left(\frac{1-n^2}{n^2+1}\right)$

Ans. 4

Sol. $|\vec{x} - \vec{y}| = n|\vec{x} + \vec{y}|$

$x^2 + y^2 - 2xy\cos\theta = n^2\left(x^2 + y^2 + 2xy\cos\theta\right)$

$x^2\left(1 + 1 - 2\cos\theta\right) = n^2x^2\left(1 + 1 + 2\cos\theta\right)$

$2\left(1 - n^2\right) = 2\cos\theta\left(n^2 + 1\right)$

$\cos\theta = \frac{1-n^2}{n^2+1}$

$\theta = \cos^{-1}\left(\frac{1-n^2}{n^2+1}\right)$

23. If combined equation of line $y = p(x)$ and $y = q(x)$ can be written as $\left(y - p(x)\right)\left(y - q(x)\right) = 0$ then angle bisector of $x^2 - 4xy - 5y^2 = 0$

1) $x^2 + 3xy + y^2 = 0$

2) $x^2 + 3xy - y^2 = 0$

3) $x^2 - 3xy + y^2 = 0$

4) $x^2 - 3xy - y^2 = 0$

Ans. 2

Sol. Equation of angle bisector of homogeneous equation of pair of straight line $ax^2 + 2hxy + by^2$ is.

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

For $x^2-4xy-5y^2=0$

$a=1, h=-2, b=-5$

So, equation of angle bisector is

$$\frac{x^2-y^2}{1-(-5)}=\frac{xy}{-2}$$

$x^2-y^2+3xy=0$

So, combined equation of angle bisector is $x^2+3xy-y^2=0$

24. Equation of a circle $i\,\mathrm{Re}\left(z^2\right)+2\left(im(z)\right)^2+2\,\mathrm{Re}(z)=0$ where $z=x+o=iy$ and a line passes through the vertex of parabola $x^2-6x+y+13=0$ and the centre of circle, then y intercept of the line is.

1) -2 2) -1 3) 2 4) 1

Ans. 2

Sol. $z=(x+iy)$

So, $z^2=x^2-y^2+i2xy$

Now $x^2-y^2+2y^2+2x=0$

$x^2+y^2+2x=0 \Rightarrow centre=(-1,0)\ and\ x^2-6x+y+13=0 (x-3)^2=-(y+4)\ (x-3)^2=-(y+4)$

$(x-3)^2=-(y+4)$

Vertex $(3,-4)$

$\therefore$ equation of line is $(y-0)=\frac{-4-0}{3+1}(x+1) \Rightarrow 4y=-4(x+1)$

$x+y+1=0 \Rightarrow \frac{x}{-1}+\frac{y}{-1}=1$

25. If $f(x)=\begin{cases}5x+1 & ;x<2\\ \int_0^x (5+|1-t|)\,dt & ;x\ge 2\end{cases}$

1) $f(x)$ is differentiable $\forall x\in R$

2) $f(x)$ continuous at $x=2$ but not differentiable at $x=2$

3) $f(x)$ continuous at $x=2$ but not differentiable at $x=1$

4) $f(x)$ is neither continuous nor differentiable at x=2

Ans. 2

Sol. $LHL=\lim_{x\to 2^-}(5x+1)=11$

$RHL=\lim_{x\to 2^+}\int_0^x (5+|1-t|)\,dt=\int_0^1 (5+(1-t))\,dt+\int_1^2 (5-(1-t))\,dt=11$

$f(2)=11$

So, $f(2)=11$

So , $f(x)$ is continuous at $x=2$

$LHD\, at x=2 is \frac{d}{dx}(5x+1)\Big|_{x=2}=5$

$RHD\, at\, x=2 is\ \frac{d}{dx}\int_0^x (5+|1-t|)\,dt\Big|_{x=2}=6$

$LHD\ne RHD$, so function is not differentiable at x=2.