

25-07-2021-SHIFT-1 PAPER-1 MATHS MEMORY BASED

1. If the ratio of coefficient of middle term in the expansion of $(1+x)^{20}$ and sum of coefficients of middle terms in the expansion of $(1+x)^{19}$ is λ , then λ is:

- 1) 2 2) 1 3) 3 4) 4

Ans. 2

Sol.
$$\frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$$

2. If $f(x) = \begin{cases} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)} & ; x < 2 \\ \mu & ; x = 2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & ; x > 2 \end{cases}$, is continuous at $x=2$. Then the sum of λ and μ is:

- 1) $e+e^2$ 2) $e-e^2$ 3) e^2-e 4) e^2+e+1

Ans. 2

Sol.
$$\text{RHL} = \lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{x-[x]}} = \lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{(x-2)}} = e$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)}$$

For $x < 2$, $|x^2 - 5x + 6| = x^2 - 5x + 6$

$$\therefore \text{LHL} = \lim_{x \rightarrow 2^-} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)} = \frac{-\lambda}{\mu}$$

Also, $f(2) = \mu$

For $f(x)$ to be continuous at $x=2$,

$$\text{RHL} = \text{LHL} = f(2)$$

$$\therefore e = \frac{-\lambda}{\mu} = \mu$$

$$\Rightarrow \mu = e \text{ and } \lambda = -e^2$$

$$\therefore \lambda + \mu = e - e^2$$

3. If $\sin x + \sin 2x + \sin 3x + \sin 4x = 0, x \in [0, 2\pi]$ then sum of values of x is

- 1) 7π 2) 9π 3) 11π 4) 12π

Ans. 2

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$$\Rightarrow 2 \sin \frac{5x}{2} \cos \frac{3x}{2} + 2 \sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

$$\Rightarrow 2 \sin \frac{5x}{2} \left(\cos \frac{3x}{2} + \cos \frac{x}{2} \right) = 0$$

$$\Rightarrow 4 \sin \frac{5x}{2} \cos x \cos \frac{x}{2} = 0$$

$$\Rightarrow \sin \frac{5x}{2} = 0 \quad \text{or} \quad \cos x = 0 \quad \text{or} \quad \cos \frac{x}{2} = 0$$

$$\Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi \quad \text{or} \quad x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \frac{x}{2} = \frac{\pi}{2}$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5} \quad \text{or} \quad x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = \pi$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \pi$$

Hence sum of all solutions = 9π

4. The number of real solutions of the equation $e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1 = 0$

- 1) 0 2) 1 3) 6 4) 8

Ans. 2

Sol. Since $f(x) = e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1$

$$\Rightarrow f'(x) = 6e^{6x} + 4e^{4x} + 6e^{3x} + 24e^{2x} + e^x > 0, \forall x \in R$$

Hence $f(x)$ is an increasing function

$$\text{Now } \lim_{x \rightarrow -\infty} f(x) = -1 \text{ and } f(0) = 1 + 1 + 2 + 12 + 1 - 1$$

$$\Rightarrow f(0) > 0$$

Hence $f(x)=0$ has a root in $(-\infty, 0)$

5. If $\frac{1}{a-b} + \frac{1}{a-2b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$ where a is so large then b such that cube and higher powers of $\frac{b}{a}$ may be neglected then value of γ is:

- 1) $\frac{b^2}{3a^3}$ 2) $\frac{a^2-b}{3a^3}$ 3) $\frac{a-b}{3a^2}$ 4) $\frac{b^2}{2a^2}$

Ans. 1

Sol.
$$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb}$$

$$= \frac{1}{a} \left[\left(1 - \frac{b}{a}\right)^{-1} + \left(1 - \frac{2b}{a}\right)^{-1} + \left(1 - \frac{3b}{a}\right)^{-1} + \dots + \left(1 - \frac{nb}{a}\right)^{-1} \right]$$

$$= \frac{1}{a} \left[\left\{ 1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 + \dots \right\} + \left\{ 1 + \left(\frac{2b}{a}\right) + \left(\frac{2b}{a}\right)^2 + \dots \right\} + \left\{ 1 + \left(\frac{nb}{a}\right) + \left(\frac{nb}{a}\right)^2 + \dots \right\} \right]$$

$$= \frac{1}{a} \left[n + \frac{b}{a}(1+2+\dots+n) + \frac{b^2}{a^2}(1^2+2^2+\dots+n^2) \right]$$

$$= \frac{1}{a} \left[n + \frac{n^2 b}{2a} + \frac{n b}{2a} + \frac{2n^3 + 3n^2 + n}{6} \left(\frac{b^2}{a^2}\right) \right]$$

$$= n \left(\frac{1}{a} + \frac{b}{2a^2} + \frac{b^2}{6a^3} \right) + \left(\frac{b}{2a^2} + \frac{b^2}{2a^3} \right) n^2 + \frac{b^2}{3a^3} n^3$$

By comparing this result to $\alpha n + \beta n^2 + \gamma n^3$

We get $\gamma = \frac{b^2}{3a^3}$

6. In an A.P., if $S_{3n} = 3S_{2n}$, then ratio $\frac{S_{4n}}{S_{2n}}$ equals:

- 1) 8 2) 6 3) 4 4) 2

Ans. 2

Sol
$$\frac{S_{3n}}{S_{2n}} = \frac{\frac{3n}{2} [2a + (3n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]} = 3$$

$$\Rightarrow 2a + (3n-1)d = 2[2a + (2n-1)d]$$

$$\Rightarrow 2a + (n-1)d = 0 \dots (1)$$

$$\text{Now } \frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2} [2a + (4n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]}$$

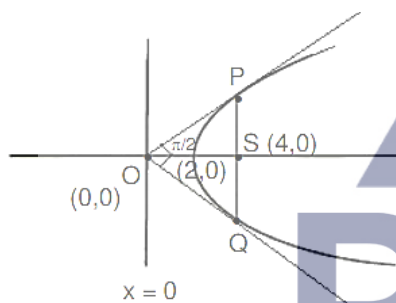
$$= \frac{2 [2a + (4n-1)d]}{[2a + (2n-1)d]}$$

$$\text{Put, } 2a = -(n-1)d, \text{ we have, } \frac{S_{4n}}{S_{2n}} = \frac{2[3nd]}{nd} = 6$$

7. A parabola whose vertex is at 2 unit distance from origin on positive x-axis and distance between focus and origin is 4 unit. The tangent drawn from the origin to the parabola meet the parabola at P and Q, then the area of ΔOPQ is

Ans. 16squnits

Sol.



Equation of parabola

$$(y-0)^2 = 4(2)(x-2)$$

Origin lie on directrix $x=0$

So ΔOPQ is right angle triangle.

Equation of chord of contact PQ is

$$T=0$$

$$X=4$$

It is latus rectum of parabola

$$\text{So area of } OPQ = \frac{1}{2} \times 4 \times 8 = 16 \text{ square unit}$$

8. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is:

1) 4

2) 120

3) 210

4) 310

Ans. 3

Sol. $\left(\left(x^{1/3}+1\right)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$$\left(x^{1/3}-x^{-1/2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \left(x^{1/3}\right)^{10-r} \left(-x^{-1/2}\right)^r$$

$$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$$

$$\Rightarrow r = 4$$

$$T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

9. The area of the region bounded by the inequalities $2x^2 < y < 4 - 2x, x > 0$ is:

1) $\frac{11}{12}$

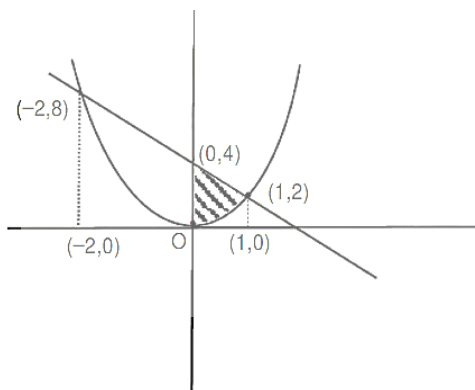
2) $\frac{7}{3}$

3) $\frac{2}{3}$

4) $\frac{5}{2}$

Ans. 2

Sol.



$$2x^2 = 4 - 2x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, x = 1$$

$$\text{Required area} = \frac{1}{2}(2+4) \times 1 - \int_0^1 2x^2 dx = 3 - \frac{2}{3} = \frac{7}{3} \text{ square units}$$

10. An ellipse with eccentricity $\frac{1}{\sqrt{3}}$ passes through the point $\left(\sqrt{\frac{3}{2}}, 1\right)$. A circle is drawn whose centre is the focus of ellipse and its radius is $\frac{2}{\sqrt{3}}$. If circle cuts the ellipse at two different points P and Q then the value of PQ^2 is

- 1) $\frac{8}{3}$ 2) $\frac{4}{3}$ 3) $\frac{16}{3}$ 4) $\frac{5}{3}$

Ans 3

Sol. Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$

$$\text{It passes through } \left(\sqrt{\frac{3}{2}}, 1\right) \Rightarrow \frac{3}{2a^2} + \frac{1}{b^2} = 1 \dots (1)$$

$$\text{Given } e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1 - e^2) = \frac{2}{3}a^2 \dots (2)$$

Solve (1) & (2) we get $a^2 = 3, b^2 = 2$

$$\therefore \text{ Ellipse is } \frac{x^2}{3} + \frac{y^2}{2} = 1 \dots (3)$$

$$\text{Focus } (\pm ae, 0) \equiv \left(\pm\sqrt{3} \cdot \frac{1}{\sqrt{3}}, 0\right) \equiv (\pm 1, 0)$$

$$\text{Hence circle is } (x-1)^2 + y^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3} \dots (4)$$

Solve (3) & (4)

$$2x^3 + 3\left(\frac{4}{3} - (x-1)^2\right) = 6$$

$$2x^2 + 4x - 3(x^2 + 1 - 2x) = 6$$

$$-x^2 + 6x - 5 = 0$$

$$x = 1, 5$$

When $x = 1 \Rightarrow \frac{1}{3} + \frac{y^2}{2} = 1 \Rightarrow y^2 = \frac{4}{3} \Rightarrow y = \pm \frac{2}{\sqrt{3}}$

Hence $P\left(\left(1, \frac{2}{\sqrt{3}}\right), Q\left(1, -\frac{2}{\sqrt{3}}\right)\right) \Rightarrow PQ^2 = \frac{16}{3}$

When $x = 5 \Rightarrow \frac{y^2}{2} = 1 - \frac{25}{3} = -\frac{22}{3} \Rightarrow$ not possible

$$PQ^2 = \frac{16}{3}$$

11. The statement $(p \rightarrow q) \wedge (p \rightarrow \neg q)$ is logically equivalent to:

1) p

2) q

3) $\neg p$

4) $\neg q$

Ans. 3

Sol. $= (p \rightarrow q) \wedge (p \rightarrow \neg q)$

$$= (\neg p \vee q) \wedge (\neg p \vee \neg q)$$

$$= \neg p \vee (q \wedge \neg q)$$

$$= \neg p \vee f$$

$$= \neg p$$

12. In class 10th, 11th and 12th of a school there are 5, 6 and 8 students respectively. The number of ways of selection of 10 students such that at least two students are selected from each of the classes and at most 5 students together can be selected from class 10th & 11th are $k \times 100$ then the value of k is.

Ans. 238

Sol.	Total student	(5)	(6)	(8)	
	Class	10 th	11 th	12 th	
		2	2	6	$\rightarrow 5_{C_2} \times 6_{C_2} \times 8_{C_6}$
		2	3	5	$\rightarrow 5_{C_2} \times 6_{C_3} \times 8_{C_5}$
		3	2	5	$\rightarrow 5_{C_3} \times 6_{C_2} \times 8_{C_5}$

$$\begin{aligned} \text{Total number of ways} &= 5_{C_2} \times 8_{C_3} (6_{C_3} + 6_{C_2}) + 5_{C_2} \times 6_{C_2} \times 8_{C_6} \\ &= 23800 \end{aligned}$$

13. If $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots + \infty\right)^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)}$ is ℓ then ℓ^2 is

Ans. 3

Sol. $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)} = \ell$

$$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_1 \frac{1}{2}} = \ell$$

Let $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots = x$

$$(x-1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \quad (1)$$

$$\frac{1}{3}(x-1) = \frac{2}{3^2} + \frac{6}{3^3} + \dots \quad (2)$$

From (1)-(2), we get

$$\frac{2}{3}(x-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$\frac{2}{3}(x-1) = \frac{2}{3} + \frac{4}{3^2} \left(\frac{1}{1 - \frac{1}{3}}\right)$$

$$\frac{2}{3}(x-1) = \frac{2}{3} + \frac{4}{3^2} \cdot \frac{3}{2}$$

$$\frac{2}{3}(x-1) = \frac{2}{3} + \frac{2}{3}$$

$$x-1 = 2 \text{ \& } x = 3$$

So, $3^{\log_1 \frac{1}{2}} = \ell$

$$3^{\frac{1}{2}} = \ell$$

$$\ell^2 = 3$$



14. A hyperbola with equation $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$ is given. A triangle is formed with two vertices as the focus of the hyperbola and third vertex lies on hyperbola. The locus of centroid of the triangle is:

1) $16(x-1)^2 - 9(y+2)^2 = 16$

2) $9(x-1)^2 - 16(y+2)^2 = 16$

3) $9(x-1)^2 + 16(y+2)^2 = 16$

4) $16(x-1)^2 + 9(y+2)^2 = 16$

Ans. 2

Sol. Given $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$

Let $x-1 = X$

$y+2 = Y$

$$\frac{X^2}{16} - \frac{Y^2}{9} = 1$$

$a = 4, b = 3$

$$b^2 = a^2(e^2 - 1) \Rightarrow \frac{9}{16} = e^2 - 1 \Rightarrow e = \frac{5}{4}$$

Focus $(\pm ae, 0) \Rightarrow X = \pm ae, Y = 0$

$x-1 = \pm 5, y+2 = 0$

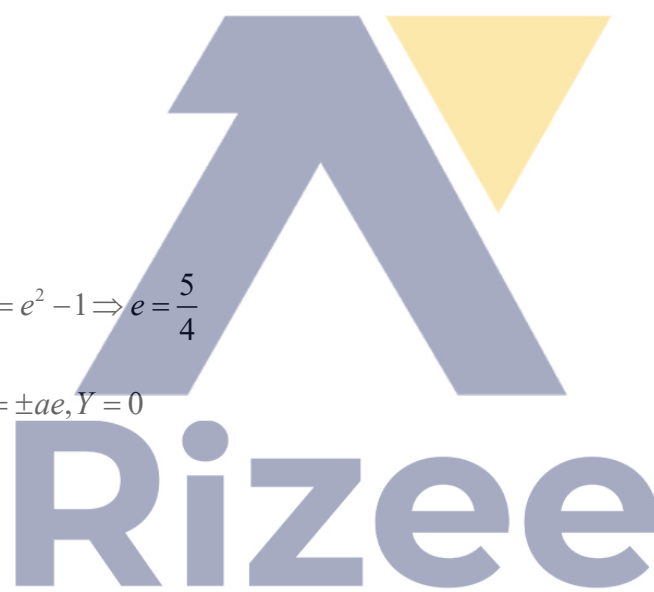
$x = 6, -4, y = -2$

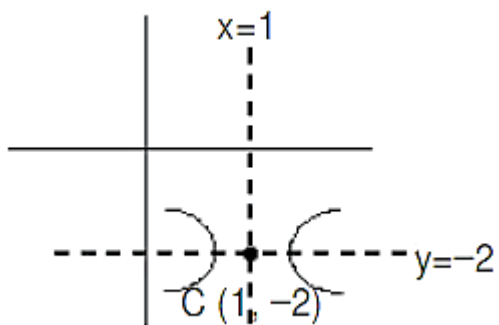
Hence focus $S(-4, -2), S'(6, -2)$

Let any point on hyperbola $x-1 = 4 \sec \theta, y+2 = 3 \tan \theta \Rightarrow P(1+4 \sec \theta, -2+3 \tan \theta)$

Hence centroid $\equiv \left(\frac{-4+6+1+4 \sec \theta}{3}, \frac{-2-2-2+3 \tan \theta}{3} \right)$

$$h = \frac{3+4 \sec \theta}{3} \Rightarrow \sec \theta = \frac{3h-3}{4}$$





$$k = \frac{-6 + 3 \tan \theta}{3} \Rightarrow \tan \theta = k + 2$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\left(\frac{3h-3}{4}\right)^2 - (k+2)^2 = 1$$

$$\text{Locus is } \frac{9(x-1)^2}{16} - \frac{(y+2)^2}{1} = 1$$

$$\Rightarrow 9(x-1)^2 - 16(y+2)^2 = 16$$

15. Evaluate $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}}$:

1) $\frac{\pi}{24}$

2) $\frac{\pi}{4}$

3) $\frac{\pi}{12}$

4) $\frac{\pi}{6}$

Ans. 3

Sol. $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\tan 2x}} \dots (1)$$

By property

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt[3]{\cot 2x}}$$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\tan 2x} dx}{1 + \sqrt[3]{\tan 2x}} \dots (2)$$

By adding (1) & (2)

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{(1 + \sqrt[3]{\tan 2x}) dx}{1 + \sqrt[3]{\tan 2x}}$$

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

16. If a set of matrix $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 1, \pm 2, \pm 3\} \right\}$ and $A \in M$ then the number of such matrices A whose determinant value is 15.

Ans. 16

Sol. $|A| = (ad - bc) = 15$

Where $a, b, c, d \in \{\pm 1, \pm 2, \pm 3\}$

Case-I $ad=9$ & $bc=-6$

$$ad \equiv (3, 3) \text{ or } (-3, -3) \quad bc \equiv (2, -3), (-2, 3), (-3, 2), (3, -2)$$

Total = $2 \times 4 = 8$ matrix

Case-II $ad=6$ and $bc=-9$

Similarly, Total = $4 \times 2 = 8$ matrix

Total such matrix = $8 + 8 = 16$ matrix

17. If $\frac{dy}{dx} = 1 + xe^{y-x}$, $-\sqrt{2} < x < \sqrt{2}$ and $y(0) = 0$ then minimum value of y is

- | | |
|---|---|
| 1) $(1 - \sqrt{3}) - \ln(\sqrt{3} - 1)$ | 2) $(1 + \sqrt{3}) - \ln(\sqrt{3} - 1)$ |
| 3) $(1 - \sqrt{3}) - \ln(\sqrt{3} + 1)$ | 4) $(1 + \sqrt{3}) - \ln(\sqrt{3} + 1)$ |

Ans. 1

Sol. $\frac{dy}{dx} = 1 + xe^{y-x} \dots (1)$

$$e^{-y} \frac{dy}{dx} = e^{-y} + xe^{-x}$$

Put $e^{-y} = t \Rightarrow e^{-y} \frac{dy}{dx} = -\frac{dt}{dx}$

$$-\frac{dt}{dx} = t + xe^{-x}$$

$$\frac{dt}{dx} + t = -xe^{-x} \dots (2)$$

$$I.F. = e^{\int 1 \cdot dx} = e^x$$

Solution of equation (2) is

$$te^x = \int (-xe^{-x}) \cdot e^x dx + c$$

$$te^x = -\frac{x^2}{2} + c$$

$$e^{x-y} = -\frac{x^2}{2} + c \dots (3)$$

$$y(0) = 0 \Rightarrow 1 = c \Rightarrow e^{x-y} = \left(\frac{2-x^2}{2} \right)$$

$$x - y = \ln \left(\frac{2-x^2}{2} \right)$$

$$y = x - \ln \left(\frac{2-x^2}{2} \right)$$

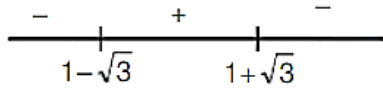
Now, $\frac{dy}{dx} = 1 + x \left(\frac{2}{2-x^2} \right)$

$$\Rightarrow \left(\frac{2-x^2+2x}{2-x^2} \right) = 0$$

$$\Rightarrow -\left(\frac{x^2-2x-2}{2-x^2} \right) = 0$$

$$x = 1 \pm \sqrt{3}$$





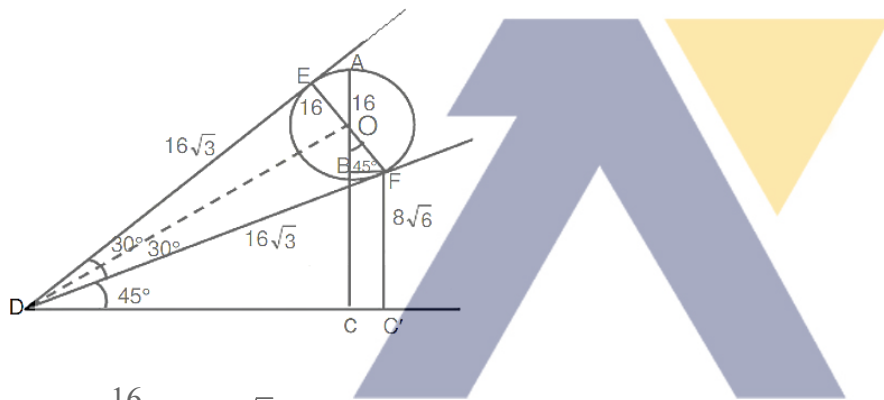
$$\Rightarrow y_{\min} \text{ at } x = 1 - \sqrt{3} \Rightarrow y_{\min} = (1 - \sqrt{3}) - \ln(\sqrt{3} - 1)$$

18. A balloon with radius 16 cm is at a certain height above the ground such that the angle of elevation of its centre is 75° from a point on the ground. If the balloon subtends an angle of 60° at that point, then the height of its topmost point from the ground is:

- 1) $8(\sqrt{6} - \sqrt{2} + 2)$ 2) $8(\sqrt{6} - 2\sqrt{2} + 2)$ 3) $8(\sqrt{6} + \sqrt{2} + 2)$ 4) $8(\sqrt{6} - 2\sqrt{2} + 2)$

Ans. 3

Sol. In triangle EOD



$$ED = \frac{16}{\tan 30^\circ} = 16\sqrt{3}$$

$$DF = 16\sqrt{3}$$

Now in $\triangle DFC'$

$$C'F = 16\sqrt{3} \cdot \sin 45^\circ$$

$$= 16\sqrt{3} \cdot \frac{1}{\sqrt{2}} = 8\sqrt{6}$$

$$CB = 8\sqrt{6}$$

In $\triangle OBF$

$$OB = 16 \sin 45^\circ$$

$$= \frac{16}{\sqrt{2}} = 8\sqrt{2}$$

$$\text{Hence of top most point} = 8\sqrt{6} + 8\sqrt{2} + 16 = (8\sqrt{6} + \sqrt{2} + 2)$$

19. If $S = \{ S = \{ n : n \in N \text{ and } 1 \leq n \leq 100, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \forall a, b, c, d \in R \}$ then the number of 2 digit number in S is

- 1) 25 2) 22 3) 24 4) 20

Ans. 2

Sol. Let $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AB = IB$$

$$(A - I)B = 0$$

$$A = I$$

$$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n = 1$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$n = \text{multiple of } 4(4, 8, \dots, 100)$

Number of two digit numbers in S = 22 (12, 16, \dots, 96)

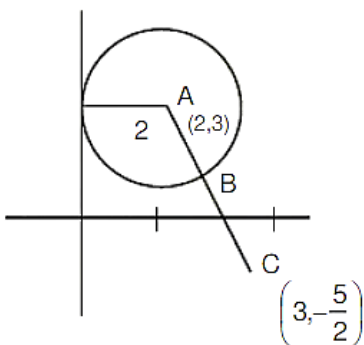
20. If $|z - (3i + 2)| < 2$ then the min value of $|2z - 6 + 5i|$ is

- 1) $(5\sqrt{5} - 2)$ 2) $(5\sqrt{5} - 4)$ 3) $(5\sqrt{5} + 2)$ 4) $\left(\frac{5\sqrt{5}}{2} + 2\right)$

Ans. 2

Sol. $|(x - 2) + (y - 3)i| < 2$

Point A(2,3) and radius=2



$$|(2x-6)+(2y+5)i|$$

$$x = 3$$

$$y = -\frac{5}{2}$$

$$\text{Point C} \left(3, -\frac{5}{2} \right)$$

$$AC = \sqrt{(3-2)^2 + \left(-\frac{5}{2}-3\right)^2}$$

$$= \sqrt{1 + \frac{121}{4}}$$

$$= \sqrt{\frac{125}{4}}$$

$$\text{Min distance } 2BC = 2 \left(\frac{5\sqrt{5}}{2} - 2 \right) = (5\sqrt{5} - 4)$$

21. Let $\vec{p} = (3\hat{i} + 2\hat{j} + \hat{k})$, $\vec{q} = (2\hat{i} + \hat{j} + \hat{k})$ and \vec{r} is perpendicular to both $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ such that $|\vec{r}| = \sqrt{3}$.
If $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k})$, then the value of $|a| + |b| + |c|$ is:

1) 0

2) 1

3) 3

4) 6

Ans. 3

Sol. $(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{r} = \pm \sqrt{3} \frac{((\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}))}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|}$$

$$= \pm (-\hat{i} + \hat{j} + \hat{k})$$

So $|a| = 1, |b| = 1, |c| = 1$

$$|a| + |b| + |c| = 3$$