JEE MAIN-2021 DATE:20-07-2021 (SHIFT-2)| PAPER-1-MATHEMATICS

1. If element of matrix A is defined as $A=\left[a_{ij}\right]_{3\times 3}$ where $A=\begin{cases}(-1)^{j-i} & i<j \\ 2 & i=j \\ (-1)^{i+j} & i>j\end{cases}$, then the value of $\left|3Adj\left(2A^{-1}\right)\right|$ is:

(1) 72 (2) 36 (3) 108 (4) 48

Ans: 3

Sol: $A=\begin{bmatrix}2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2\end{bmatrix}$

So, $|A|=\begin{vmatrix}2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2\end{vmatrix}$

$=2(4-1)+1(-2+1)+1(1-2)$

$=2(3)+1(-1)+1(-1)$

$=4$

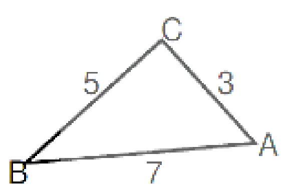
$\left|3Adj\left(2A^{-1}\right)\right|=3^3\left|Adj\left(2A^{-1}\right)\right|=3^3\times\left|2A^{-1}\right|^2$

$=3^3\times 2^6\times\left|A^{-1}\right|^2=3^3\times 2^6\times\frac{1}{|A|^2}=108$

2. In a ΔABC, if $\left|\overrightarrow{AB}\right|=7,\left|\overrightarrow{BC}\right|=5$, and $\left|\overrightarrow{CA}\right|=3$. If projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $\left(\frac{n}{2}\right)$, then the value of n is:

Ans: 05.00

Sol: $\left|\overrightarrow{AB}\right|=7,\left|\overrightarrow{BC}\right|=5,\left|\overrightarrow{CA}\right|=3$,



Projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $=\left|\overrightarrow{BC}\right|\ \cos\angle BCA$

$5\left|\left(\frac{3^2+5^2-7^2}{2.3.5}\right)\right|==5\left|\frac{-15}{30}\right|=\frac{5}{2}$

3. The value of $\tan\left(2\tan^{-1}(3/5)+\sin^{-1}(5/13)\right)$ is:

(1) $\frac{220}{21}$ (2) $\frac{110}{21}$ (3) $\frac{55}{21}$ (4) $\frac{20}{11}$

Ans: 1

Sol: $\tan\left(\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}}+\tan^{-1}\frac{5}{12}\right)$

$$\tan\left(\tan^{-1}\left(\frac{15}{8}\right)+\tan^{-1}\left(\frac{5}{12}\right)\right)=\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}}=\frac{220}{21}$$

4. Mean of 6 observations is 10 and their variance is $\frac{20}{3}$. If observations are 15,11,10,7,a,b then $|a-b|$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 4

Ans: 2

Sol: Mean=10

$$\frac{7+10+11+15+a+b}{6}=10$$

$a+b=17\ldots.(1)$

Variance $=\frac{20}{3}$

$$\frac{49+100+121+225+a^2+b^2}{6}-100=\frac{20}{3}$$

$a^2+b^2=145\ldots.(2)$

$(a+b)^2=289$

$ab=72$

$(a-b)^2=(a+b)^2-4ab$

$(a-b)^2=289-288=1$

$|a-b|=1$

5. If $f(x)=x+1$, then find $\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+\ldots+f\left(\frac{5(n-1)}{n}\right)\right]$

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Ans: 1

Sol: $=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)=\int_0^1 f(5x)\,dx=\int_0^1(5x+1)\,dx$

$$=\left(\frac{5x^2}{2}+x\right)_0^1=\frac{5}{2}+1=\frac{7}{2}$$

6. Sum of 21 terms of series $\log_{9^{1/2}}x+\log_{9^{1/3}}x+\log_{9^{1/4}}x+\ldots$ is 252, then the value of x is:

(1) 7 (2) 243 (3) 9 (4) 81

Ans: 3

Sol: $2\log_9 x+3\log_9 x+4\log_9 x\ldots 21\text{ terms}$

$$=(2+3+4+5\ldots+22)\log_9 x=\frac{21}{2}(2+22)\log_9 x$$

$= 21 \times 12 \log_9 x$

$= 252 \log_9 x$

Given sum $= 252 \Rightarrow \log_9 x = 1$

$\Rightarrow x = 9$

7. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left([x]-[\sin x]\right) dx = ?$ (where [.] represents G.I.F.)

(1) -2 (2) 1 (3) 0 (4) -1

Ans: 3

Sol: $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left([x]+[-x]\right) dx$

Using property $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(-x)\,dx$

$I = \int^{\frac{\pi}{2}} \left([x]+[-x]\right) dx - \int_{0}^{\frac{\pi}{2}} \left([\sin x]+[-\sin x] dx\right) = 0$

8. If $\lim_{x\to 0} \frac{\alpha x e^{x} - \beta \ln(1+x) + \gamma x^2 e^{-x}}{x^3} = 10$, then the value of $\alpha+\beta+\gamma$ is :

Ans: 3

Sol: $\lim_{x\to 0} \frac{\alpha x\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....\right) - \beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}+.....\right) \gamma x^2\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+....\right)}{x^3} = 10$

$\Rightarrow \alpha - \beta = 0, \Rightarrow \alpha = \beta$

$\Rightarrow \alpha + \frac{\beta}{2} + \gamma = 0 \Rightarrow = -\frac{3\beta}{2}$

$\Rightarrow \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$

$\Rightarrow \frac{\beta}{2} - \frac{\beta}{3} + \frac{3\beta}{2} = 10 \Rightarrow \frac{3\beta - 2\beta + 9\beta}{6} = 10$

$\because \beta = 6, \alpha = 6, \gamma = -9$

So the value of $\alpha + \beta + \gamma = 3$

9. The value of x satisfying the equation $\log_{(x+1)}\left(2x^2+7x+5\right) + \log_{(2x+5)}(x+1)^2 = 4$ is:

(1) -2 (2) 2 (3) -4 (4) 4

Ans: 2

Sol: $\log_{(x+1)}\left((2x+5)(x+1)\right) + \log_{(2x+5)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$

Put $\log_{(x+1)}(2x+5) = t$

$\therefore 1+t+\frac{2}{t}=4$

$t^2+t+2=4t \Rightarrow t^2-3t+2=0$

$t=1,\ t=2$

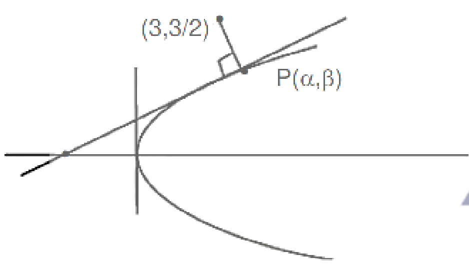
For $t=1$	For $t=2$
$2x+5=x+1$	$2x+5=(x+1)^2$
$\Rightarrow x=-4$ (rejected)	$x=2,\ x=-2$ (rejected)

10. If (α,β) is the point on $y^2=6x$, that is closest to $\left(3,\frac{3}{2}\right)$ then find $2(\alpha+\beta)$

(1) 6 (2) 9 (3) 7 (4) 5

Ans: 2

Sol:



$y^2=6x$

$2yy'=6$

$\frac{dy}{dx}=\frac{3}{\beta}$

$-\frac{\beta}{3}=\frac{\beta-3/2}{\alpha-3}$

$-\frac{\beta}{3}=\frac{2\beta-3}{2\alpha-6}$

$-\beta(2\alpha-6)=6\beta-9$

$6\beta-2\alpha\beta=6\beta-9$

$\alpha\beta=\frac{9}{2}\Rightarrow\beta=\frac{9}{2\alpha}$

$\therefore \beta^2=6\alpha$

$\frac{81}{4\alpha^2}=6\alpha$

$\alpha^3=\frac{27}{8}\alpha=\frac{3}{2},\beta^2=9\Rightarrow\beta=\pm3$

$\alpha=\frac{3}{2},\beta=3$

$2(\alpha+\beta)=9$

11. Two circles pass through (-1,4) and their centres lie on $x^2+y^2+2x+4y=4r$. If r_1 and r_2 are maximum 4 minimum radii and $\frac{r_1}{r_2}=a+b\sqrt{2}$ then the value of $a+b$ is

Ans: 3

Sol: Given circle

$$(x+1)^2+(y+2)^2=(3)^2$$

Any point on this circle is $(3\cos\theta-1,3\sin\theta2)$ equation of circle having centre $(3\cos\theta-1,3\sin\theta-2)$

$$r=\sqrt{(3\cos\theta-1+1)^2+(3\sin\theta-2-4)^2}$$

$$=\sqrt{9\cos^2\theta+9\sin^2\theta+36-36\sin\theta}$$

$$\sqrt{45-36\sin\theta}$$

$$\Rightarrow \frac{r_1}{r_2}=9=r_1 \text{ and } r_{\min}=3=r_2$$

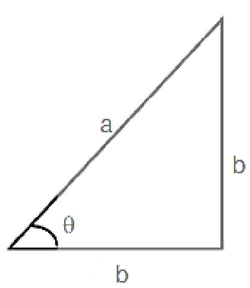
$$\Rightarrow a+b=3$$

12. If ΔABC is right angled triangle with sides a,b & c and smallest angle θ. If $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ are also the sides of right angled triangle then find $\sin\theta$

(1) $\sqrt{\frac{3-\sqrt{5}}{2}}$ (2) $\frac{3-\sqrt{5}}{2}$ (3) $\sqrt{\frac{3+\sqrt{5}}{2}}$ (4) $\frac{3+\sqrt{5}}{2}$

Ans: 1

Sol: Let a>b>c



$$\sin\theta=\frac{c}{a}$$

$$\frac{1}{a}<\frac{1}{b}<\frac{1}{c}$$

$$\frac{1}{c^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

$$1=\frac{c^2}{a^2}+\frac{c^2}{b^2}$$

$$1=\frac{c^2}{a^2}+\frac{c^2}{a^2+c^2}\text{[As } a^2=b^2+c^2\text{]}$$

$$1=\sin^2\theta+\frac{1}{\frac{a^2}{c^2}-1}=\sin^2\theta+\frac{1}{\cos ec^2\theta-1}$$

$$1=\frac{1-\sin^2\theta+1}{\cos ec^2\theta-1}\Rightarrow \sin^2 q+\cos ec^2\theta=3$$

13. If $\text{Re}\left[\left(1+\cos\theta+2i\sin\theta\right)^{-1}\right]=4$ then value of θ is:

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $-\frac{\pi}{2}$ (4) π

Ans: 4

Sol: $\frac{1}{1+\cos^2\theta+2i\sin\theta}\times\frac{1+\cos\theta-2i\sin\theta}{1+\cos\theta-2i\sin\theta}$

$=\frac{1+\cos\theta-2i\sin\theta}{\left(1+\cos\theta\right)^2+4\sin^2\theta}$

$\Rightarrow\frac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4\sin^2\theta}=4$

$\Rightarrow\frac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4-4\cos^2\theta}=4$

$\Rightarrow\frac{1+\cos\theta}{5+2\cos\theta-3\cos^2\theta}=4$

$\Rightarrow 1+\cos\theta=20+8\cos\theta-12\cos^2\theta$

$\Rightarrow 1+\cos\theta-7\cos\theta-19=0$

$\Rightarrow 12\cos^2\theta-7\cos\theta-19=0$

$\Rightarrow 12\cos^2\theta-19\cos\theta+12\cos\theta-19=0$

$\Rightarrow \cos\theta\left(12\cos\theta-19\right)+1\left(12\cos\theta-19\right)=0$

$\Rightarrow \cos\theta=-1$ or $\cos\theta=\frac{19}{12}\left(rejected\right)$

$\Rightarrow \theta=\pi$

14. If $x=ay-1=z-2$, and $x=3y-2=bz-2$ lies in same plane then the value of a,b, is

(1) a=2,b=3 (2) a=1,b=1 (3) b=1, $a\in R-\{0\}$ (4) $a=3,b=2$

Ans: 3

Sol: $\frac{x}{1}=\frac{y-\frac{1}{a}}{\frac{1}{a}}=\frac{z-2}{1}, x=\frac{y-\frac{2}{3}}{\frac{1}{3}}=\frac{z-\frac{2}{b}}{\frac{1}{b}}$

$\left(\vec{a}_1-\vec{a}_2\right).\left(\vec{b}_1\times\vec{b}_2\right)=0$

$$\begin{vmatrix} 0 & \frac{1}{a}-\frac{2}{3} & 2-\frac{2}{b} \\ 0 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix}=0$$

$\Rightarrow\frac{1}{ab}-\frac{1}{a}=0$

$b=1, a\in R-\{0\}$

15. If $P\left(\overline{A}\cap B\right)+P\left(A\cap\overline{B}\right)=1-K$

$P(\overline{A}\cap C)+P(A\cap\overline{C})=1-2K$

$P(\overline{B}\cap C)+P(\overline{B}\cap\overline{C})=1-K$

$P(A\cap B\cap C)=K^2, K\in(0,1)$

Then the value of P(at least one of A,B,C) is:

(1) $>\frac{1}{2}$ (2) $\left[\frac{1}{8},\frac{1}{4}\right]$ (3) $<\frac{1}{4}$ (4) $\frac{1}{4}$

Ans: 1

Sol: $P(A)+P(B)-2P(A\cap B)=1-K$

$P(A)+P(C)-2P(A\cap C)=1-2K$

$P(B)+P(C)-2P(B\cap C)=1-K$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

$=\frac{3-4k}{2}+k^2=\frac{2k^2-4k+3}{2}$

$\because$ The value of $2k^2-4k+3$ is greater than 1

$\therefore\ P(A\cup B\cup C)>\frac{1}{2}$

16. If $f(x)=\frac{5x+3}{6x+a}$ and $f(f(x))=x$ then the value of a is:

(1) -5 (2) 5 (3) 6 (4) -6

Ans: 1

Sol: $f(f(x))=\frac{5f(x)+3}{6f(x)+a}=x\Rightarrow 5f(x)+3=6xf(x)+ax$

$\Rightarrow\frac{25x+15}{6x+a}+3=6x\left(\frac{5x+3}{6x+a}\right)+ax$

$\Rightarrow 25x+15+18x+3a=30x^2+18x+6ax^2+a^2x$

$\Rightarrow(30+6a)x^2+(a^2-25)x-(3a+15)=0$

$\Rightarrow 6(a+5)x^2+(a-5)(a+5)x-3(a+5)=0,\qquad\forall x$

$\Rightarrow a+5=0\Rightarrow a=-5$

17. If $g(t)=\begin{cases}\max(t^3-6t^2+9t-3,0) & , t\in[0,3]\\ 4-t & , t\in(3,4]\end{cases}$ then the number of points at which g(t) is non differentiable is:

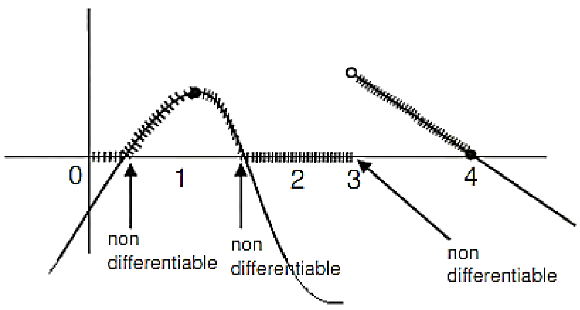
(1) 1 (2) 3 (3) 2 (4) 4

Ans: 2

Sol: $y=t^3-6t^2+9t-3$

$y'=3t^2-12t+9$

$= 3\left(t^2 - 4t + 3\right)$



18. $A: \text{if } 2+4=7, \text{ then } 3+4=8$

B: if 2+4=7, then 3+4=8

C: if A and B are true, then 5+4=11

(1) A is true, B and C are false

(2) B is true, A and C are false

(3) C is true, A and B are false

(4) B is false, A and C are true

Ans: 4

Sol: Truth table $p \to q$

P	Q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

A is true, B is false, C is true.

19. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B \sum_{r=1}^{2021} A^r$ then value of $|B|$ is

(1) 2021

(2) $(2021)^2$

(3) -2021

(4) 0

Ans: 2

Sol: A=I, B=I+I+......2021 times

$$B = 2021\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$|B| = (2021)^2$$