

JEE MAIN-2021 DATE:20-07-2021 (SHIFT-1)| PAPER-1-MATHEMATICS

1. All possible word with or without meaning were formed using all the letters of the word 'EXAMINATION'. The probability that 'M' appears at fourth position is:

(1) $\frac{2}{11}$

(2) $\frac{1}{11}$

(3) $\frac{4}{11}$

(4) $\frac{8}{11}$

Ans: 2

Sol: EXAMINATION

$E \rightarrow 1$ $n(S) = \frac{11!}{2!2!2!}$

$X \rightarrow 1$ $n(E) = \frac{10!}{2!2!2!}$

$A \rightarrow 2$ $P(E) = \frac{n(E)}{n(s)} = \frac{1}{11}$

$M \rightarrow 1$

$O \rightarrow 1$

$T \rightarrow 1$

$N \rightarrow 2$

$I \rightarrow 2$

2. If a cricket team consist of 15 players have 6 batsmen, 7 Ballers and 2 wicket keepers then the number of ways in which cricket team formed with atleast 4 batsmen, 5 ballers and 1 wicket keeper

(1) 567

(2) 525

(3) 462

(4) 777

Ans: 4

Sol: Case-I: Team consist 5 Batsman, 5 Bowlers and 1 wicket keeper then number of ways.

$$= {}^6C_5 \times {}^7C_5 \times {}^2C_1 = 6 \times 21 \times 2 = 252$$

Case-II: 4 Batsmen, 6 blowers and 1 wicket keeper

$$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 = 15 \times 7 \times 2 = 210$$

Case-III: 4 Batsmen, 5 bowler and 2 wicket keepers

$$= {}^6C_4 \times {}^7C_5 \times {}^2C_2 = 15 \times 21 \times 1 = 315$$

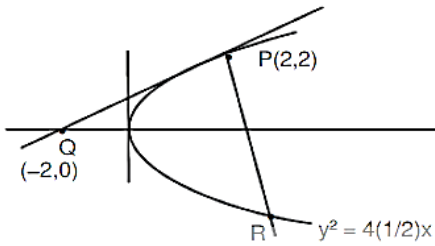
$$\text{Total } 252 + 210 + 315 = 777$$

3. Tangent drawn at a point P(2,2) to parabola $y^2 = 2x$ cuts x-axis at point Q and normal drawn at point P(2,2) to parabola cut parabola again at point R then area of ΔPQR is

- (1) 25 (2) $\frac{25}{2}$ (3) $\frac{15}{2}$ (4) 50

Ans: 2

Sol:



Equation of tangent at P(2,2) is T=0

$$2y = x + 2$$

$$y^2 = 4 \text{ So, } Q(-2,0)$$

$$2at_1 = 1 \Rightarrow t_1 = 2$$

$$t_2 = -t_1, -\frac{2}{t_1} = -2 - \frac{2}{2} = -3$$

$$\therefore R\left(\frac{1}{2}(-3)^2, 2\left(\frac{1}{2}\right)(-3)\right) = \left(\frac{9}{2}, -3\right)$$

$$\text{Area of } \Delta PQR = \frac{1}{2} \begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [2(0+3) - 2(-2-9/2) + 1(6-0)] = \frac{1}{2} [6+4+9+6] = \frac{25}{2} \text{ sq.unit}$$

4. Coefficient of X^{256} in the expansion of $(1-x)^{101} (x^2+x+1)^{100}$ is

- (1) ${}^{100}C_{86}$ (2) ${}^{100}C_{85}$ (3) ${}^{100}C_{84}$ (4) ${}^{100}C_{83}$

Ans: 2

$$\text{Sol: } \Rightarrow (1-x)^{101} (x^2+x+1)^{100}$$

$$\Rightarrow (1-x)^{100} (x^2+x+1)^{100} (1-x)$$

$$\Rightarrow (1-x^3)^{100} (1-x)$$

$$\Rightarrow (1-x) \left({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 + \dots + {}^{100}C_{84}x^{252} - {}^{100}C_{85}x^{255} + {}^{100}C_{86}x^{256} + \dots \right)$$

$$\Rightarrow {}^{100}C_{85}x^{256}$$

So, the coefficient of x^{256} is ${}^{100}C_{85}$

5. The value of $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\frac{x^2+2}{x}}$ is

Ans: 1

Sol: $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\frac{x^2+2}{x}}$ (1^∞ form)

$$= e^{\lim_{x \rightarrow 0} \frac{(1 - \cos x \sqrt{\cos 2x})}{x} (x^2 + 2)}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos^2 x (\cos 2x)}{x} \right\} (x^2 + 2)}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos^2 x (\cos 2x)}{x} \right\} \left(\frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}} \right)}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos^2 (2 \cos^2 x - 1)}{x} \right\} \left(\frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}} \right)}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos^2 x (\cos 2x)}{x} \right\} \left(\frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}} \right)}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos^2 (2 \cos^2 x - 1)}{x} \right\} \left(\frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}} \right)}$$

$$= e^{\lim_{x \rightarrow 0} \frac{(1 - 2 \cos^4 x + \cos^2 x)}{x} \frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{-(2 \cos^4 x - \cos^2 x - 1)}{x} \frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{(2 \cos^2 x + 1)(\cos^2 - 1)}{x} \frac{x^2 + 2}{1 + \cos \sqrt{\cos 2x}}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{+(2 \cos^2 x + 1) \sin^2 x}{x} \frac{x^2 + 2}{1 + \cos x \sqrt{\cos 2x}}}$$

$$= e^{\lim_{x \rightarrow 0} (2 \cos^2 x + 1) \frac{\sin x}{x} \cdot \sin x \frac{x^2 + 2}{1 + \cos \sqrt{\cos 2x}}}$$

$$= e^0 = 1$$

6. If the focal chord $y = mx + c$ of parabola $y^2 = -64x$ is also the tangent to the circle $(x+10)^2 + y^2 = 4$ then absolute value of $4\sqrt{2}(m+c)$ is

Ans: 34

Sol: Focus of parabola is $(-16,0)$

$$\text{So, } -16m + c = 0 \Rightarrow c = 16m \dots (i)$$

Now slope form of tangent to the circle

$(x+10)^2 + y^2 = 4$ is given by

$$y = m(x+10) \pm 2\sqrt{1+m^2}$$

$$\text{So, } c = 10m \pm 2\sqrt{1+m^2} \dots (ii)$$

By (i) and (ii)

$$16m = 10m \pm 2\sqrt{1+m^2}$$

$$\Rightarrow 9m^2 = 1 + m^2 \Rightarrow m = \pm \frac{1}{2\sqrt{2}}$$

$$\Rightarrow c = 16m = \pm \frac{8}{\sqrt{2}}$$

$$\therefore 4\sqrt{2}(m+c) = \pm 34$$

7. The mean of 6 numbers is 6.5 and its variance is 10.25 if 4 numbers are 2,4,5 and 7; then find the other two:

(1) 10,11

(2) 11,12

(3) 9,12

(4) 9,11

Ans: 1

Sol: Let two number x and y according to question

$$18 + x + y = 39$$

$$x + y = 21 \dots (1)$$

$$10.25 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$10.25 = \frac{x^2 + y^2 + 4 + 16 + 25 + 49}{6} - (6.5)^2$$

$$10.25 = \frac{x^2 + y^2 + 94}{6} - (6.5)^2$$



$$\Rightarrow x^2 + y^2 = 221 \dots (2)$$

Solving (1) and (2)

SO, $x = 10$ or $y = 11$

8. A continuous & differentiable function $f(x)$ is increasing in $\left(-\infty, \frac{3}{2}\right)$ and decreasing in $\left(\frac{3}{2}, \infty\right)$ then

$x = \frac{3}{2}$ is:

(1) Point of local maxima

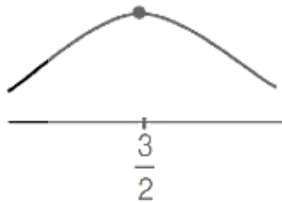
(2) Point of local minima

(3) Point of Inflection

(4) None of these

Ans: 1

Sol: Roughly graph of $f(x)$ can be drawn as



Thus $x = \frac{3}{2}$ is a point of local maxima

9. If the roots of the quadratic equation $x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$ are α and β then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$

(1) 50.3^{24}

(2) 51.3^{24}

(3) 52.3^{24}

(4) 104.3^{24}

Ans: 3

Sol: $x^2 + \sqrt{3} = -3^{\frac{1}{4}}x$

$$\Rightarrow x^4 + 2\sqrt{3}x^2 + 3 = \sqrt{3}x^2$$

$$\Rightarrow x^4 + \sqrt{3}x^2 + 3 = 0$$

$$\Rightarrow x^8 + 6x^4 + 9 = 3x^4$$

$$\Rightarrow x^8 + 3x^4 + 9 = 0$$

$$\Rightarrow \alpha^8 = -9 - 3\alpha^4$$

$$\Rightarrow \alpha^{12} = -9\alpha^4 - 3\alpha^8 = -9\alpha^4 - 3(-9 - 3\alpha^4) = 27$$

Similarly $\beta^{12} = 27$

$$\Rightarrow \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) = (27)^8 \cdot 26 + (27)^8 \cdot 26 = 52 \cdot (27)^8 = 52 \cdot 3^{24}$$

Ans: 3

$$\text{Sol: Shortest distance} = \frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$$
$$\Rightarrow 9 \frac{|((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})|}{\sqrt{64 + 64 + 16}}$$

$$\Rightarrow \left| \frac{8(\alpha + 4) + 16 + 12}{12} \right| = 9$$

$$\therefore \alpha = 6$$

13. If $\vec{a}, \vec{b}, \vec{c}$ are mutually \perp unit vectors equally inclined to $\vec{a} + \vec{b} + \vec{c}$ at an angle θ , find $36 \cos^2 2\theta$

Ans: 4

$$\text{Sol: } |\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 3$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$

$$\text{Now } \vec{a}(\vec{a} + \vec{b} + \vec{c}) = |\vec{a}| |\vec{a} + \vec{b} + \vec{c}| \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{3}} \Rightarrow \cos 2\theta = 2 \cos^2 \theta - 1$$

$$\Rightarrow \cos 2\theta = -\frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{1}{9} \Rightarrow 36 \cos^2 2\theta = 4$$

14. If z and ω are complex number such that $|z\omega| = 1, \arg(z) - \arg(\omega) = \frac{3\pi}{2}$. Find the $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 2\bar{z}\omega}\right)$.

(1) $\frac{\pi}{4}$

(2) $-\frac{\pi}{4}$

(3) $\frac{3\pi}{4}$

(4) $-\frac{3\pi}{4}$

Ans: 4

$$\text{Sol: Let } z = re^{i\theta} \text{ \& } \omega = \frac{1}{r} e^{i\left(\theta - \frac{3\pi}{2}\right)}$$

$$\text{then } \frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega} = \frac{1 - 2re^{-i\theta} \cdot \frac{1}{r} e^{i\left(\theta - \frac{3\pi}{2}\right)}}{1 + 3re^{-i\theta} \cdot \frac{1}{r} e^{i\left(\theta - \frac{3\pi}{2}\right)}}$$

$$= \frac{1 - 2e^{i\frac{3\pi}{2}}}{1 + 3e^{-i\frac{3\pi}{2}}} = \frac{1 - 2i}{1 + 3i}$$

$$= -\frac{1}{2} - \frac{1}{2}i$$

$$\text{The arg}\left(-\frac{1}{2} - \frac{1}{2}i\right) = -\frac{3\pi}{4}$$

15. If $f(x) = \begin{cases} \sin x - e^x & ; x \leq 0 \\ a + [-x] & ; 0 < x < 1 \\ 2x - b & ; x \geq 1 \end{cases}$ is continuous and differentiable function then find the value of $a + b$.

(where $[\cdot]$ is GIF)

Ans: (03.00)

Sol: Since $f(x)$ is continuous at $x=0$

$$\text{So } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$-1 = a - 1 = -1 \Rightarrow a = 0$$

Since $f(x)$ is continuous at $x=1$

$$\text{So } \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$$a - 1 = 2 - b = 2 - b$$

$$\Rightarrow a = 0, \text{ so } 0 - 1 = 2 - b$$

$$\Rightarrow -3 = -b$$

$$\Rightarrow b = 3$$

So the value of $a + b = 3$



16. If $A = [a_{ij}]_{3 \times 3}$ where $a_{ij} = \begin{cases} 1 & i = j \\ -x & |i - j| = 1 \\ 2x + 1 & \text{otherwise} \end{cases}$ and $f(x) = \det(A)$, then the sum of local maximum and

local minimum value of $f(x)$ is:

(1) $\frac{20}{27}$

(2) $\frac{-20}{27}$

(3) $\frac{88}{27}$

(4) $\frac{-88}{27}$

Ans: 4

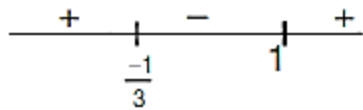
$$\text{Sol: } |A| = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 1 + x^2(2x+1) + x^2(2x+1) - (2x+1)^2 - x^2 - x^2$$

$$\Rightarrow f(x) = 4x^3 - 4x^2 - 4x$$

$$\Rightarrow f'(x) = 12x^2 - 8x - 4$$

$$= 4(3x^2 - 2x - 1)$$

$$= 4(x-1)(3x+1)$$



$\Rightarrow f(x)$ is maximum at $x = \frac{-1}{3}$ and minimum at $x=1$

\therefore maximum value $= \frac{20}{27}$ and minimum value $= -4$

$$\therefore \text{sum} = \frac{20}{27} - 4 = -\frac{88}{27}$$

17. The coefficient of $a^3b^4c^5$ in $(ab+bc+ac)^6$ is:

(1) 60

(2) 45

(3) 40

(4) 90

Ans: 1

Sol: $(ab+bc+ac)^6 = \sum_{p+q+r=6} \frac{6!}{p!q!r!} (ab)^p (bc)^q (ca)^r$

$$\sum_{p+q+r=6} \frac{6!}{p!q!r!} a^{p+r} b^{p+q} c^{q+r}$$

For $a^3b^4c^5$, we need

$$p+r=3$$

$$p+q=4$$

$$q+r=5$$

Solving we get, $p=1, q=3, r=2$

\therefore coefficient of $a^3b^4c^5$ in $(ab+bc+ac)^6$ is $\frac{6!}{1!2!3!} = 60$

18. If an invertible function $f(x)$ is defined as $f(x)=3x-2$, $g(x)$ is also an invertible function such that $f^{-1}(g^{-1}(x))=x-2$ then $g(x)$ is

(1) $\frac{x-8}{3}$

(2) $\frac{x+8}{3}$

(3) $\frac{x-3}{8}$

(4) $\frac{x+3}{8}$

Ans: 2

Sol: $f^{-1}(g^{-1}(x))=x-2$

$$f(x-2)=g^{-1}(x)$$

$$3(x-2)-2 = g^{-1}(x)$$

$$3x-8 = g^{-1}(x)$$

$$g^{-1}(x) = 3x-8$$

$$\text{Or } x = 3g(x)-8$$

$$g(x) = \frac{x+8}{3}$$

19. $\int_{-1}^1 \ln(\sqrt{1-x} + \sqrt{1+x}) dx = ?$

- (1) $\pi + \ln 2$ (2) $2 \ln 2$ (3) $\frac{\pi}{2} - 1 + \ln 2$ (4) $\ln 2 - \frac{\pi}{2} - 1$

Ans: 4

Sol: $f(x) = \ln(\sqrt{1-x} + \sqrt{1+x})$ $x \in [-1, 1]$ is an even function

$$\Rightarrow I = 2 \int_0^1 \ln(\sqrt{1-x} + \sqrt{1+x}) dx$$

Put $x = \cos 2\theta \Rightarrow dx = -2 \sin 2\theta d\theta$

$$\therefore \cos \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta d\theta$$

$$\Rightarrow I = -4 \int_{\frac{\pi}{4}}^0 \left[\ln \left\{ (\sin \theta + \cos \theta) \sqrt{2} \right\} \right] \sin 2\theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{4}} \ln \left\{ (\sin \theta + \cos \theta) \sqrt{2} \right\} \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \ln(\sin \theta + \cos \theta) \sin 2\theta d\theta + 4 \ln \sqrt{2} \int_0^{\frac{\pi}{4}} \sin 2\theta d\theta$$

$$= 4 \left[-\ln(\sin \theta + \cos \theta) \frac{\cos 2\theta}{2} \Big|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} -\frac{\cos \theta - \sin \theta}{\sin \theta + \cos \theta} \cdot \frac{\cos 2\theta}{2} d\theta \right] + 4 \ln \sqrt{2} \left(-\frac{\cos 2\theta}{2} \right) \Big|_0^{\frac{\pi}{4}}$$

$$= 4 \left[0 - \frac{1}{2} \int_0^{\frac{\pi}{4}} (\cos \theta - \sin \theta)^2 d\theta \right] + 4 \ln \sqrt{2} \left(0 + \frac{1}{2} \right)$$

$$\begin{aligned}
&= 4 \left[0 - \frac{1}{2} \int_0^{\frac{\pi}{4}} (1 - \sin 2\theta) d\theta \right] + 2 \ln \sqrt{2} \\
&= -2 \left[\theta + \frac{\cos 2\theta}{2} \right]_0^{\frac{\pi}{4}} + \ln 2 \\
&= -2 \left[\frac{\pi}{4} - \frac{1}{2} \right] + \ln 2 = -\frac{\pi}{2} - 1 + \ln 2
\end{aligned}$$

20. The probability selecting integers $a \in [-5, 30]$, such that $x^2 + 2(a+4)x - 5a + 64 > 0$ for all $x \in R$ is :

- (1) $\frac{7}{9}$ (2) $\frac{4}{9}$ (3) $\frac{2}{9}$ (4) $\frac{1}{3}$

Ans: 3

Sol: $x^2 + 2(a+4)x - (5a - 64) > 0$

$$D < 0$$

$$\therefore 4(a+4)^2 + 4(5a - 64) < 0$$

$$\Rightarrow (a+4)^2 + (5a - 64) < 0$$

$$\Rightarrow a^2 + 13a - 48 < 0$$

$$a = \frac{-13 \pm \sqrt{169 + 192}}{2} = -16, 3$$

So, $a \in (-16, 3)$

So, $a = -5, -4, -3, -2, -1, 0, 1, 2$

$$\therefore \text{Required Probability} = \frac{8}{36} = \frac{2}{9}$$

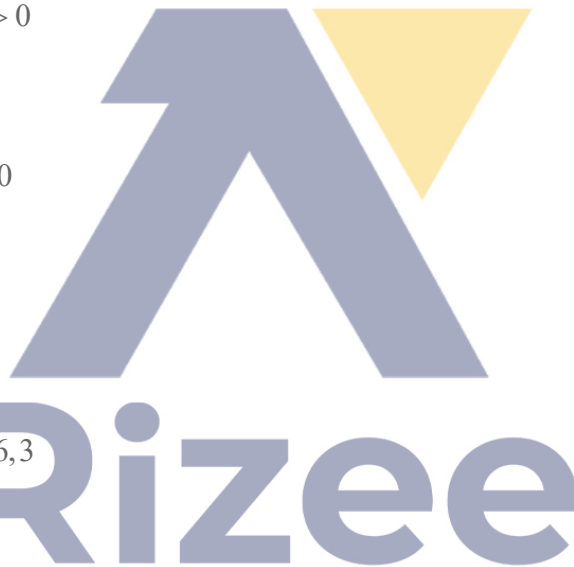
21. If $\int_0^a e^{x-[x]} dx - 10e - 9$, then the value of 'a' is (where $[x]$ is GIF)

- (1) $9 + \ln 2$ (2) $10 + \ln 2$ (3) 10 (4) 9

Ans: 2

Sol: Let $a = 10 + K$, $0 \leq K < 1$

$$\int_0^a e^{\{x\}} dx = 10e - 9$$



$$\int_0^{10} e^{\{x\}} dx + \int_{10}^{10+k} e^{\{x\}} dx = 10e - 10 + 1 \Rightarrow \int_{10}^{10+k} e^{\{x\}} dx = 1 \Rightarrow \int_0^K e^{\{x\}} dx = 1$$

$$e^K - 1 = 1$$

$$k = \ln 2$$

$$\text{So, } a = 10 + \ln 2$$

22. If $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|$ then $|\vec{A} - \vec{B}|$ is

(1) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

(2) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

(3) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

(4) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

Ans: 4

Sol: $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| \Rightarrow \cos \theta = \sin \theta \Rightarrow \tan \theta = 1$

$$\theta = \frac{\pi}{4}$$

$$|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2\vec{A} \cdot \vec{B} \quad |\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2\vec{A} \cdot \vec{B}$$

$$= A^2 + B^2 - 2AB \cos\left(\frac{\pi}{4}\right)$$

$$= A^2 + B^2 - \sqrt{2}AB$$

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

