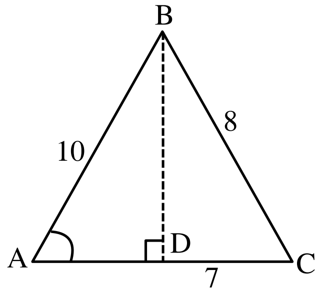
1. In a ΔABC $|\overrightarrow{AB}| = 10$, $|\overrightarrow{BC}| = 8$, $|\overrightarrow{CA}| = 7$, then projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is

(1) $\frac{72}{73}$ (2) $\frac{85}{73}$ (3) $\frac{85}{14}$ (4) $\frac{14}{85}$

Ans. (3)

Sol. $\cos A = \frac{10^2 + 7^2 - 8^2}{2.10.7}$



Projection of AB on AC = 10 cos A

$\Rightarrow 10 \cdot \frac{85}{140} = \frac{85}{14}$

2. If 15 $\sin^4 \theta + 10 \cos^4 \theta = 6$ then find the value of 8 $\text{cosec}^6 \theta + 27 \sec^6 \theta$

(1) 150 (2) 250 (3) 350 (4) 450

Ans. (2)

Sol. $15 \sin^4 \theta + 10 \cos^4 \theta = 6$

$\Rightarrow 15 \sin^4\theta + 10 (1 - \sin^2 \theta)^2 = 6$

$\Rightarrow 25 \sin^4 \theta - 20 \sin^2\theta + 4 = 0$

$\Rightarrow (5 \sin^2 \theta - 2)^2 = 0 \Rightarrow \sin^2\theta = \frac{2}{5}, \cos^2 \theta = \frac{3}{5}$

Now $27 \text{ cosec}^6 \theta + 8 \sec^6 \theta = 27\left(\frac{125}{27}\right) + 8\left(\frac{125}{8}\right) = 250$

3. In a series of '2n' observation, half are (a) and other half are (–a) if 'b' is added in all the observation then the mean and S.D. of new series are '5' and '20' respectively then find the value of $a^2 + b^2$

(1) 450 (2) 480 (3) 475 (4) 425

Ans. (4)

Sol. Given series

(a, a, a .... n times), (–a, –a, –a ...... n times)

now $\bar{x} = \frac{\sum x_i}{2n} = 0$

as $x_i \to x_i + b$

then $\bar{x} \to \bar{x} + b$

So, $\bar{x} + b = 5 \Rightarrow b = 5$

no. change in S.D. due to change in origin

$\sigma = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \sqrt{\frac{2na^2}{2n} - 0}$

$20 = \sqrt{a^2} \Rightarrow a = 20$

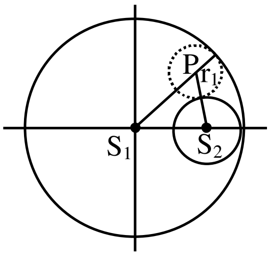
$a^2 + b^2 = 425$

4. Let the circle are $S_1 : x^2 + y^2 = 9$, $S_2 : (x - 2)^2 + y^2 = 1$. The locus of the center of the circle which touches S_1 internally and S_2 externally always passes through a point

(1) (3, 0) (2) $(\sqrt{3}, 1)$ (3) (4, 0) (4) (–3, 0)

Ans. (1)

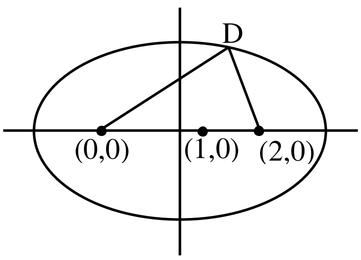
Sol. $PS_1 = 3 - r_1$



$PS_2 = 1 + r_1$

$PS_1 + PS_2 = 4$

so locus is ellipse & $2a = 4 \Rightarrow a = 2$ & $2ae = 2$



Equation of Ellipse is $\frac{(x-1)^2}{2^2} + \frac{y^2}{3} = 1 \quad \Rightarrow e = \frac{1}{2}$

$\Rightarrow b^2 = 3$

(3, 0) satisfied it

5. If $g(x) = \int_0^x f(t)dt$ a here f(x) is a continuous function on [0,3] such that $\forall$ x $\in$ [0,1], f(x) has ranges $\left[\frac{1}{3},1\right]$ & $\forall$ x $\in$ (1,3] , f(x) has ranges $\left[0,\frac{1}{2}\right]$. Then the maximum range in which g(3) can lie is

(1) $\left[\frac{1}{3},2\right]$ (2) (1,2) (3) $\left[\frac{5}{6},3\right]$ (4) $\left[1,\frac{4}{3}\right]$

Ans. (1)

Sol. $\int_0^1 \frac{1}{3}dt + \int_1^3 0.dt < g(3) < \int_0^1 1.dt + \int_1^3 \frac{1}{2}dt$

$\frac{1}{3} < g(3) < 2$

6. An experiment is performed 5 times. The probability of 1 success is 0.4096 & probability of 2 success is 0.2048, find the probability of 3 success.

(1) $\frac{32}{125}$ (2) $\frac{16}{125}$ (3) $\frac{16}{25}$ (4) $\frac{32}{225}$

Ans. (1)

Sol. ${}^5C_1 . p^1 . q^4 = 0.4096$

$\Rightarrow 5pq^4 = 0.4096$ ....(i)

${}^5C_2 . p^2 . q^3 = 0.2048$

$\Rightarrow 10p^2q^3 = 0.2048$ ....(ii)

(i) ÷ (ii) $\Rightarrow \frac{q}{2p} = 2 \Rightarrow q = 4p$

$p + q = 1 \Rightarrow p = \frac{1}{5}, q = \frac{4}{5}$

p (3 sum) $= {}^5C_3 \left(\frac{1}{5}\right)^3 . \left(\frac{4}{5}\right)^2 = 10 \times \frac{1}{125} \times \frac{16}{25} = \frac{16\times2}{125} = \frac{32}{125}$

7. Let S_n denotes sum of first 'n' terms of an A.P. such that $S_{4n} - S_{2n} = 1000$, then the value of S_{6n} is

(1) 1000 (2) 3000 (3) 5000 (4) 7000

Ans. (2)

Sol. $S_{4n} - S_{2n} = 1000$

$\Rightarrow \frac{4n}{2}(2a + (4n-1)d) - \frac{2n}{2}(2a + (2n-1)d) = 1000$

$\Rightarrow 2an + 6n^2d - nd = 1000$

$\Rightarrow \frac{6n}{2}(2a + (6n-1)d) = 3000$

$\therefore S_{6n} = 1000$

8. Which of following is a tautology

(1) $((p \to q) \wedge \sim q) \to p \wedge q$ (2) $((p \to q) \wedge \sim q) \to p$

(3) $((p \to q) \wedge \sim q) \to q$ (4) $((p \to q) \wedge \sim q) \to \sim q$

Ans. (4)

Sol. (A) $\{(\sim p \vee q) \wedge \sim q\} \to (p \wedge q)$

$= (\sim p \wedge \sim q) \to (p \wedge q)$

$= \sim (p \vee q) \to (p \wedge q)$

$= (p \vee q) \vee (p \wedge q)$

$= (p \vee q)$

(B) $(p \vee q) \vee p \equiv p \vee q$

(C) $(p \vee q) \vee q \equiv p \vee q$

(D) $(p \vee q) \vee \sim p \equiv t$

9. If system of equation $4x - \lambda y + 2z = 0$, $2x + 2y + z = 0$, $\mu x + 2y + 3z = 0$, has non trivial solution than.

(1) $\lambda = 6, \mu = 2$ (2) $\mu = 6, \lambda \in R$ (3) $\mu = 5, \lambda \in R$ (4) None of these

Ans. (2)

Sol. $$\begin{vmatrix} 4 & -\lambda & 2 \\ 2 & 2 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 4(4) + l(6 - \mu) + 2(4 - 2\mu) = 0$

$\Rightarrow 16 + 6\lambda - \lambda\mu + 8 - 4\mu = 0$

$\Rightarrow \lambda = 6, \mu \in R$

10. Find the area bounded by the curve $4y^2 = x^2(x - 4)(2 - x)$

(1) $\frac{3\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{3\pi}{2}$ (4) $\frac{3\pi}{4}$

Ans. (3)

Sol. $4y^2 = x^2(x - 4)(2 - x)$

is defined for $x \in [2, 4] \cup \{0\}$

$2|y| = |x|\sqrt{(x-4)(2-x)} = |x|\sqrt{-x^2+6x-8}$

$x \ge 0$

$2|y| = x\sqrt{1-(x-3)^2}$

$$A = -2\int_2^4 \frac{x}{2}\sqrt{1-(x-3)^2}$$

$$= -\int_2^4 (6-2x-6)\sqrt{1-(x-3)^2}\,dx = -\int_2^4 (6-2x)\sqrt{1-(x-3)^2}\,dx + 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$$

$$= \left[\left(\frac{2}{3}\left((x-4)(2-x)\right)^{3/2}\right)_2^4 + 3\frac{x-3}{2}\sqrt{(x-4)(2-x)} + \frac{1}{2}\sin^{-1}(x-3)\right]_2^4$$

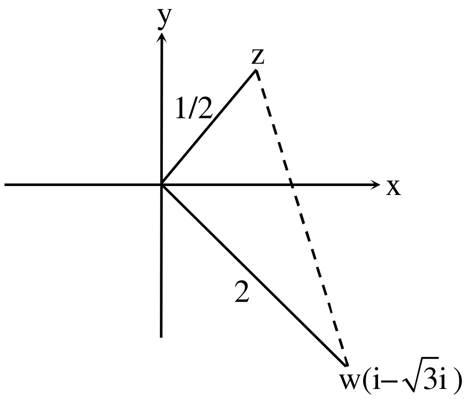
$$A = \frac{3\pi}{2}$$

11. Let $\omega = 1 - \sqrt{3}\,i$; $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then area of the triangle formed by the points w, z and origin is

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

Ans. (2)

Sol.



Area of $\Delta woz = \frac{1}{2} \times 2 \times \frac{1}{2}$

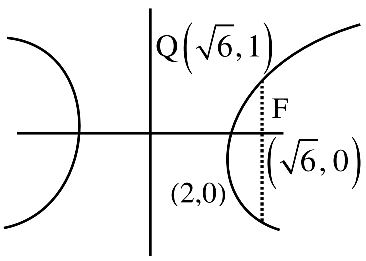
$= \frac{1}{2}$

12. Let the curve is $x^2 - 2y^2 = 4$. Tangent drawn at P $\left(4, \sqrt{6}\right)$ cuts the x-axis at R. and latus rectum at $Q(x_1, y_1)$ $(x_1 > 0)$, F be focus neartest to P. Area of ΔQPF

(1) $2 - \frac{\sqrt{6}}{7}$ (2) $2 - \frac{\sqrt{6}}{2}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{6}}{2}$

Ans. (2)

Sol. $\frac{x^2}{4} - \frac{y^2}{2} = 1$



$e^2 = 1 + \frac{2}{4} = \frac{3}{2}$

$\Rightarrow e = \sqrt{\frac{3}{2}}$

$F(\sqrt{6}, 0)$, $Q(\sqrt{6}, 1)$, $P(4, \sqrt{6})$

Area $(\Delta QPF) = \frac{4 - \sqrt{6}}{2}$

13. If $\frac{dy}{dx} = (y + 1)\left[(y+1)e^{\frac{x^2}{2}} - x\right]$, y(0) = 2, then y′(1) is equal to

(1) $\frac{15}{4\sqrt{e}}$ (2) $\frac{13}{2\sqrt{e}}$ (3) $\frac{15}{7\sqrt{e}}$ (4) $\frac{17}{4\sqrt{e}}$

Ans. (1)

Sol. $\frac{dy}{dx} + (y + 1)\, x = (y + 1)^2\, e^{\frac{x^2}{2}}$

$(y + 1)^{-2} \frac{dy}{dx} + (y + 1)^{-1}\, x = e^{\frac{x^2}{2}}$

Put $(y + 1)^{-1} = z \Rightarrow -\frac{dy}{(y+1)^2} = dz$

$\Rightarrow -\frac{dz}{dx} + zx = e^{\frac{x^2}{2}} \Rightarrow \frac{dz}{dx} + (-x)\, z = -e^{\frac{x^2}{2}}$

I.F. $= e^{\int -x\, dx} = e^{-\frac{x^2}{2}}$

Solution is $\left(e^{-\frac{x^2}{2}}\right) z = \int -e^{\frac{x^2}{2}} \times e^{-\frac{x^2}{2}}\, dx$

$\Rightarrow \left(e^{-\frac{x^2}{2}}\right)(y + 1)^{-1} = -x + c$

$\frac{e^0}{2+1} = -0 + c \Rightarrow c = \frac{1}{3}$

at x = 1, $\frac{e^{-\frac{1}{2}}}{y+1} = -1 + \frac{1}{3} = -\frac{2}{3}$

$\Rightarrow$ at x = 1, $y + 1 = -\frac{3}{2}\, e^{-\frac{1}{2}}$

Now $\left(-\frac{3}{2}e^{-\frac{1}{2}}\right)^{-2} y'(1) + \left(-\frac{2}{3}\right)e^{1/2} = e^{1/2}$

$\Rightarrow \frac{4}{9}ey'(1) = \frac{5}{3}\, e^{1/2} \Rightarrow y'(1) = \frac{15}{4\sqrt{e}}$

14. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is :

(1) 4 (2) 120 (3) 210 (4) 310

Ans. (3)

Sol. $\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\;(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \quad \Rightarrow \quad 20 - 2r - 3r = 0$

$\Rightarrow \quad r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

15. Let A and B are two square matrix of order n. A relation R is defined such that $R = \{(A, B) | A = P^{-1}BP$ for some invertible matrix P$\}$, then R is

(1) equivalence
(2) reflexive only
(3) symmetric only
(4) transitive only

Ans. **(1)**

Sol. for reflexive

$(A, A) \in R \quad \Rightarrow \quad A = P^{-1}AP$

which is true for P = I

$\therefore$ reflexive

for symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1}BP \quad \Rightarrow \quad PA = PP^{-1}BP \quad \Rightarrow \quad PAP^{-1} = IBPP^{-1}$

$\Rightarrow \quad PAP^{-1} = IBI \quad \Rightarrow \quad PAP^{-1} = B \Rightarrow \quad B = PAP^{-1}$

$\therefore$ $(B, A) \in R$ for matrix P^{-1} $\qquad \therefore$ R is symmetric

for transitivity

$A = P^{-1}BP$ and $B = P^{-1}CP \Rightarrow \quad A = P^{-1}(P^{-1}CP)P$

$\Rightarrow \quad A = (P^{-1})^2\,CP^2 \quad \Rightarrow \quad A = (P^2)^{-1}\,C(P^2)$

$\therefore$ $(A, C) \in R$ for matrix P^2 $\qquad \therefore$ R is transitive

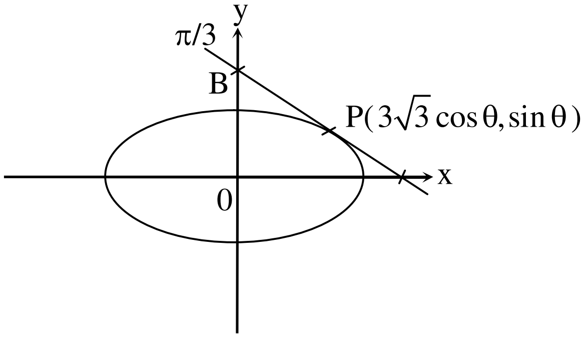
so R is equivalence

16. If tangent at a point $(3\sqrt{3}\cos\theta, \sin\theta)$ on a curve $\frac{x^2}{27} + y^2 = 1$ cuts intercept on coordinate axis then find the value of 'θ' for which the sum of intercept is minimum

(1) $\frac{\pi}{2}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{4}$

Ans. (3)

Sol.



Equation of tangent

$$\frac{x}{3\sqrt{3}}\cos\theta + y\sin\theta = 1$$

$$A\left(\frac{3\sqrt{3}}{\cos\theta}, 0\right), B\left(0, \frac{1}{\sin\theta}\right)$$

Now sum of intercept = $\frac{3\sqrt{3}}{\cos\theta} + \frac{1}{\sin\theta}$

$y = 3\sqrt{3}\ \sec\theta + \text{cosec}\,\theta$

$y' = 3\sqrt{3}\ \sec\theta\tan\theta - \text{cosec}\theta\cot\theta$

$y' = 0 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

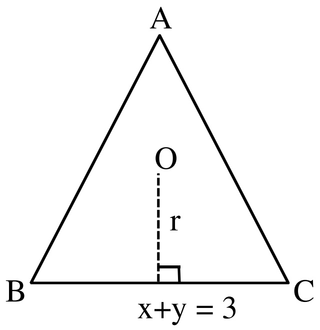
$\theta = \frac{\pi}{6}$

17. Centroid of an equilateral ΔABC is origin and one of its side is x + y = 3 then the value of R + r is

(1) $\frac{9}{\sqrt{2}}$ (2) $\frac{7}{\sqrt{2}}$ (3) $\frac{5}{\sqrt{2}}$ (4) $5\sqrt{2}$

Ans. (1)

Sol. $r = \frac{3}{\sqrt{2}}$



$R = 2r$

$R + r = 3r = \frac{9}{\sqrt{2}}$

18. Let $f : R - \{3\} \to R\ \{1\}$; $f(x) = \frac{x-2}{x-3}$ and $g : R \to R$; $g(x) = 2x - 3$ and $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$.

Then sum of all values of x is

(1) 2 (2) 3 (3) 5 (4) 7

Ans. (3)

Sol. $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$

$\Rightarrow 2(3x - 2) + (x - 1)(x + 3) = 13(x - 1)$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x = 2$ or 3

19. If $xdy - y\,dx = \sqrt{x^2 - y^2}\,dx$, $y(1) = 0$ and the area bounded by $y = f(x)$, $y = 0$, $x = 1$ and $x = e^{\pi}$ is $(\alpha e^{2\pi} + \beta)$ find $10(\alpha + \beta)$.

Ans. 4.0

Sol. $xdy - ydx = \sqrt{x^2 - y^2}\,dx \Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x}\sqrt{1 - \frac{y^2}{x^2}}\,dx \Rightarrow \int \frac{d\left(\frac{x}{y}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n|x| + c$

at $x = 1$, $y = 0 \Rightarrow c = 0$

$y = x \sin(\ell nx)$

$A = \int_1^{e^{\pi}} x \sin(\ell nx)\,dx$

$x = e^t$, $dx = e^t dt = \int_0^{\pi} e^{2t} \sin(t)\,dt$

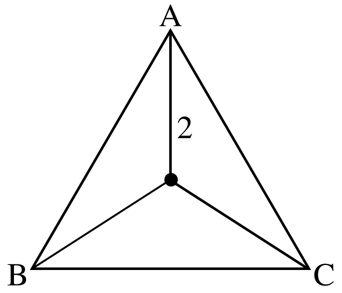
$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \frac{1 + e^{2\pi}}{5}$

$\alpha = \frac{1}{5}$, $\beta = \frac{1}{5}$ so $10(\alpha + \beta) = 4$

20. In a ΔABC whose circum radius is 2. A pole standing inside the ΔABC and angle of elevation of top of the pole from points A, B, C is 60° then find height of pole

Ans. 3.464

Sol.



$\tan 60° = \frac{h}{2} \Rightarrow h = 2\sqrt{3}$

21. Let $Q(x) = f(x^3) + xg(x^3)$ and $Q(x)$ is divisible by $x^2 + x + 1$, then find value of $Q(1)$

Ans. 0

Sol. roots of $x^2 + x + 1$ are ω and ω^2 now

$Q(\omega) = f(1) + \omega g(1) = 0$

$Q(\omega^2) = f(1) + \omega^2 g(1) = 0$

add $\Rightarrow 2\,f(1) - g(1) = 0$; $g(1) = 2f(1) \Rightarrow f(1) = g(1) = 0$

$Q(1) = f(1) + g(1) = 0 + 0 = 0$

22. If $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ and $P^n = 5I = 8P$ then value of n is

Ans. 6

Sol. $P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

$P^4 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}$

$P^6 = \begin{bmatrix} -4 & 3 \\ -15 & 11 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

and

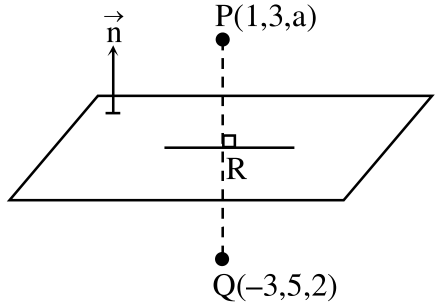
$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - 8\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$

So n = 6

23. Image of point (1, 3, a) in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-b=0$ is (–3, 5, 2) find $|a+b|$

Ans. 1

Sol.



plane : $2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10$ .... (i)

$<PQ> = <4, -2, a-2>$

$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2} \Rightarrow a - 2 = 2 \Rightarrow a = 4, b = -3$

$\therefore |a + b| = 1$

24. Let $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin(2x)}{2(x)} & x < 0 \\ b & x = 0 \\ \frac{\sqrt{x - bx^3} - \sqrt{x}}{bx^{5/2}} & x > 0 \end{cases}$, f(x) is continuous at x = 0, then $|a + b|$ is equal to

Ans. 4.5

Sol. LHL $= \frac{(a+1)}{2} + 1 = b = f(0)$

$\Rightarrow 2b = a + 3$ ....(i)

RHL $= \lim_{x \to 0^+} \frac{\sqrt{1 - bx^2} - 1}{bx^2}$

$= \lim_{x \to 0^+} \frac{-bx^2}{bx^2\left(\sqrt{1 - bx^2} + 1\right)}$

$= -\frac{1}{2}$

$\therefore b = -\frac{1}{2} \Rightarrow a = -4$

$|a + b| = \frac{9}{2}$

25. If $\sum_{k=0}^{10}(2^k+3)\,{}^{10}C_k = \alpha.2^{10} + \beta.3^{10}$ then value of $\alpha + \beta$

Ans. 4

Sol. $\sum_{k=0}^{10} {}^{10}C_k 2^k + \sum_{k=0}^{10} 3.\ {}^{10}C_k$

$3^{10} + 2.2^{10} \Rightarrow \alpha = 3, \beta = 1 \Rightarrow \alpha + \beta = 4$

26. Let f(x) is a differentiable function $\forall$ x $\in$ R such that f(x + y) = f(x) . f(y) and f'(0) = 3 then $\lim_{x\to 0}\frac{f(x)-1}{x}$ is equal to

Ans. 3

Sol. If f(x + y) = f(x) . f(y) & f'(0) = 3 then

$f(x) = a^x \Rightarrow f'(x) = a^x \ell n\, a$

$\Rightarrow f'(0) = \ell n\, a = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$$\lim_{x\to 0}\frac{f(x)-1}{x} = \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right) = 1 \times 3 = 3$$

27. Let f(x) be a cubic polynomial such that it has maxima at x = –1, minima at x = 1, $\int_{-1}^{1} f(x)\,dx = 18$, f(2) = 10 find sum of coefficient of f(x).

Ans. 8

Sol. f'(x) = k (x+ 1)(x –1)

$\therefore\ f(x) = \frac{kx^3}{3} - kx + C$

$f(2) = 10 = \frac{8k}{3} - k + C$ …(i)

$$\int_{-1}^{1} f(x)\,dx = 18 \Rightarrow \int_{-1}^{1}\left(k\left(\frac{x^3}{3}-x\right)+C\right)dx = 18$$

$\Rightarrow 0 + 2C = 18 \Rightarrow C = 9$

$\therefore\ 1 = \frac{2k}{3} \Rightarrow k = \frac{3}{2}$ …(from (i))

$\therefore\ f(x) = \frac{x^3}{2} - \frac{3}{2}x + 9$

Sum of co-efficient = –1 + 9 = 8