1. Let circle

$C_1 :\ x^2 + y^2 = 1$

$C_2 :\ x^2 + y^2 - 2y = 1$

$C_3 :\ x^2 + y^2 - 2x = 1$

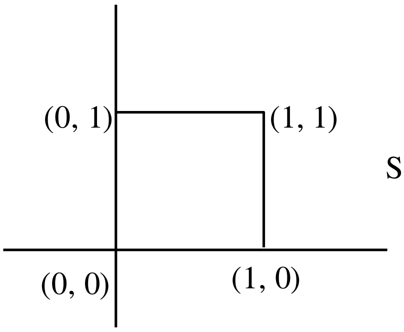
$C_4 :\ x^2 + y^2 - 2x - 2y + 1 = 0$

figure formed by joining centres of C_1, C_2, C_3, C_4 is

(1) square (2) rectangle (3) rhombus (4) parallelogram

Ans. (1)

Sol. Centres (0, 0), (1, 0), (0, 1), (1, 1)



Square

2. The values of a and b for which the function $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$ is differentiable, are respectively

(1) $\frac{1}{2}, \frac{3}{2}$ (2) $-\frac{1}{2}, \frac{3}{2}$ (3) $\frac{1}{2}, -\frac{3}{2}$ (4) 1, – 2

Ans. (2)

Sol. f(x) is continuous at x = 1 $\Rightarrow$ 1 = a + b

f(x) is differentiable at x = 1 $\Rightarrow -1 = 2a$

$\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$

3. Find the sum of all the four digit numbers formed by using the digits 1,2,2,3.

(1) 36664 (2) 26664 (3) 24 (4) 46632

Ans. (2)

Sol. ....1 = $\frac{3!}{2!}$

...........2 = 3!

...........3 = $\frac{3!}{2!}$

sum of digits at unit's place = 3 + 12 + 9 = 24

sum of all four digits numbers = 24(1111) = 26664

4. Find the number of points of intersection of the curves :

$S_1 : x^2 + y^2 - 22x + 10y + 137 = 0$ &

$S_2 : x^2 + y^2 - 10x - 10y + 41 = 0$

(1) 1 (2) 2 (3) 0 (4) 3

Ans. (1)

Sol. $S_1 : (x - 11)^2 + (y - 5)^2 = 9 \rightarrow C_1 = (11, 5)$

$S_2 : (x - 5)^2 + (y - 5)^2 = 9 \rightarrow C_2 = (5, 5)$

$r_1 = 3$ & $r_2 = 3$

$d(C_1C_2) = \sqrt{(11-5)^2} = 6$

$r_1 + r_2 = 6$

$\therefore$ Circles touch externally

Hence, 1 point of intersection.

5. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots + a_{40}x^{40}$ then the value of $a_1 + a_2 + \ldots.. + a_{37}$ is :

(1) $2^{18}(2^{19} - 19)$ (2) $2^{19}(2^{20} - 21)$ (3) $2^{18}(2^{20} - 38)$ (4) $2^{20}(2^{20} - 21)$

Ans. (2)

Sol. Put x = 1, – 1and subtract

$4^{20} - 2^{20} = (a_0 + a_1 + \ldots.. + a_{40}) - (a_0 - a_1 + \ldots\ldots.)$

$\Rightarrow 4^{20} - 2^{20} = 2\,(a_1 + a_3 + \ldots.. + a_{39})$

$\Rightarrow a_1 + a_3 + \ldots.. + a_{37} = 2^{39} - 2^{19} - a_{39}$

$a_{39} =$ coeff of x^{39} in $(1 + x + 2x^2)^{20} = {}^{20}C_1\, 2^{19}$

$\Rightarrow a_1 + a_3 + \ldots\ldots + a_{37} = 2^{39} - 2^{19} - 20(2^{19})$

$= 2^{39} - 21\,(2^{19}) = 2^{19}(2^{20} - 21)$

6. Find the sum of the series :

$$S = \frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \ldots\ldots. + \frac{1}{201^2 - 1}$$

(1) $\frac{1}{4}$ (2) $\frac{25}{101}$ (3) $\frac{26}{99}$ (4) $\frac{26}{102}$

Ans. (2)

Sol. $$S = \sum_{r=1}^{100} \frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \frac{1}{(2r+1)\,.\,2(r)}$$

$$\therefore \quad S = \frac{1}{4}\sum_{r=1}^{100}\left[\frac{1}{r} - \frac{1}{r+1}\right]$$

$$S = \frac{1}{4}\left(\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots\ldots + \left(\frac{1}{100} - \frac{1}{101}\right)\right)$$

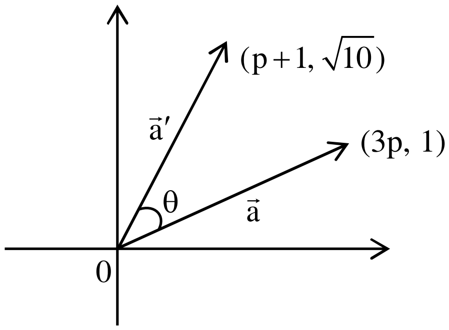
$$\therefore \quad S = \frac{1}{4}\left[\frac{100}{101}\right] = \frac{25}{101}$$

7. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesion system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components (p + 1) and $\sqrt{10}$, then find the value of 'p'

(1) $-\frac{5}{4}$ (2) $\frac{4}{5}$ (3) 1 (4) –1

Ans. (4)

Sol.



given $\vec{a} = 3p\hat{i} + \hat{j}$, $\vec{a} = (p+1)\hat{i} + \sqrt{10}\hat{j}$

$|\vec{a}| = |\vec{a}'|$, (No Change in magnitude)

$\Rightarrow \sqrt{9p^2+1} = \sqrt{(p+1)^2+10}$

$9p^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0$

$p = -1, p = \frac{5}{4}$

8. The value of $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+4}}{\sqrt{4x^2-4x+5}}dx$ is

(1) $\frac{1}{2}\sin\left(\sqrt{4x^2-4x+5}\right)+C$ (2) $\sin\left(\sqrt{4x^2-4x+5}\right)+C$

(3) $\tan\left(\sqrt{4x^2-4x+5}\right)+C$ (4) None of these

Ans. (1)

Sol. Put $\tan^{-1}\left(\frac{2x-1}{2}\right) = \theta$

$\Rightarrow 2x - 1 = 2\tan\theta$

$\Rightarrow I = \int \frac{2\tan\theta\cos(2\sec\theta)}{2\sec\theta}.\sec^2\theta d\theta$

$\Rightarrow I = \int \tan\theta\sec\theta\cos(2\sec\theta)d\theta$

Put 2 sec θ = t ⇒ 2 secθ tanθ dθ = dt

$$\Rightarrow I = \frac{1}{2}\int \cos t \; dt = \frac{\sin t}{2} + c = \frac{\sin(2\sec\theta)}{2} + C$$

$$= \frac{1}{2}\sin\left(\sqrt{4x^2 - 4x + 5}\right) + C$$

9. Let $a|z|^2 + \bar{\alpha}z + \bar{z}\alpha + d = 0$ represents the equation of the circle when

(1) $\alpha^2 - ad > 0$ such that $a \in R$

(2) $\alpha^2 - ad \geq 0$ such that $a \in R - \{0\}$

(3) $\alpha^2 - ad = 0$ for $\forall\ a \in R$

(4) $\alpha^2 - ad \leq 0$ for $\forall\ a \in R$

Ans. (2)

Sol. $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0$ ..... (i)

$$z\bar{z} + \frac{\alpha}{a}\bar{z} + \frac{\bar{\alpha}}{a}z + \frac{d}{a} = 0 \Rightarrow \text{circle}$$

$$\text{centre} = -\frac{\alpha}{a},\ r = \sqrt{|\alpha|^2 - c}$$

$$\Rightarrow \left|\frac{\alpha}{a}\right|^2 - \frac{d}{a} \geq 0 \text{ for equation (i) to represents a circle}$$

$$\Rightarrow |\alpha|^2 - ad \geq 0$$

10. Let the system of equation $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$, $\gamma u + \alpha v + \beta w = 0$ has non trial solution and α, β, γ are distinct roots of $x^3 + ax^2 + bx + c = 0$ find value of $\frac{a^2}{b}$

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (1)

Sol.
$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \alpha^2 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

$\therefore$ $\alpha + \beta + \gamma = 0$ or $\alpha = \beta = \gamma$

Here $\alpha + \beta + \gamma = 0$

$\therefore$ (α, β, γ distinct)

$\therefore$ $a = 0$

11. Let $f(x) = \sqrt{x}$ and g(x) $\sqrt{1-x}$ and fucntion

$f(x) + g(x), f(x) - g(x), \frac{f(x)}{g(x)}, \frac{g(x)}{f(x)}, g(x) - f(x)$

Find the set of value of 'x' which is common to the domain of all the five functions.

(1) $x \in (0,1)$ (2) $x \in [0,1]$ (3) $x \in [0,1)$ (4) $x \in (0,1]$

Ans. (1)

Sol. $D_f = [0, \infty)$

$D_g = (-\infty,1]$

$D_{f+g} = D_{f-g} = D_{g-f} = [0,\infty) \cap (-\infty,1] = [0,1]$

for $\frac{f}{g}$ and $\frac{g}{f}$ to be defined, $x \neq 0,1 \Rightarrow$ common domain = (0,1)

12. If $(100)^{\alpha} - (199)^{\beta} = (100)(100) + (99)(101) + (98)(102) + \ldots.. + (1)(199)$. Then find the slope of line joining (α, β) and origin.

(1) 540 (2) 550 (3) 530 (4) 545

Ans. (2)

Sol. RHS = $\sum_{r=0}^{99}(100-r)(100+r)$

$= (100)^3 - \frac{99\times100\times199}{6} = (100)^3 - (1650)\ 199$

LHS = $(100)^{\alpha} - (199)\ \beta$

So, $\alpha = 3, \beta = 1650$

Slope = $\tan\theta = \frac{\beta}{\alpha}$

$\tan\theta = 550$

13. Let the straight lines are y = (mx + 1), 3x + 4y = 9. Find number of integral values of m for which abscissa of point of intersection is integer

(1) 1 (2) 2 (3) 3 (4) 0

Ans. (2)

Sol. $3x + 4(mx + 1) = 9$

$x(3 + 4m) = 5$

$x = \frac{5}{(3+4m)}$

$(3 + 4m) = \pm1, \pm5$

$4m = -3 \pm 1, -3 \pm 5$

$4m = -4, -2, -8, 2$

$m = -1, -\frac{1}{2}, -2, \frac{1}{2}$

Two integral value of m

14. Find the differential equation of the curve $y^2 = 4a(x + a)$ where a is any arbitrary constant.

(1) $y(y')^2 + 2xy' - y = 0$ (2) $y(y')^2 - 2xy' + y = 0$

(3) $y(y')^2 + 2xy' + y = 0$ (4) $y(y')^2 - 2xy' - y = 0$

Ans. (1)

$y^2 = 4a(x + a)$ ...... (i)

$2yy' = 4a$

$\therefore\ yy' = 2a$

$\therefore$ by (i) $y^2 = 2yy'\left(x + \frac{yy'}{2}\right)$

$y^2 = 2yy'x + (yy')^2$

$\Rightarrow y(y')^2 + 2xy' - y = 0$

(as $y \neq 0$)

15. If $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$, $(x \in (0, \pi))$

Then the values of 'x' which satisfy this are :

(1) $\frac{\pi}{12}, \frac{5\pi}{12}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$ (3) $\frac{2\pi}{3}, \frac{\pi}{3}$ (4) $\frac{\pi}{9}, \frac{2\pi}{9}$

Ans. (1)

Sol. $R_1 \to R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$\therefore\ 2 + 8\sin 2x - 4\sin 2x = 0$

$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$

16. If $2A + B = \begin{bmatrix} 0 & 0 & 3 \\ 10 & 1 & 9 \\ -1 & 4 & 0 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 0 & -5 & 9 \\ -5 & 3 & 2 \\ -3 & 2 & 0 \end{bmatrix}$

Find the value of $(t_r(A) - t_r(B))$.

(1) 1 (2) – 2 (3) – 1 (4) 2

Ans. (4)

Sol. $t_r(2A + B) = 1 \Rightarrow 2t_r(A) + t_r(B) = 1$ .....(1)

$t_r(A - 2B) = 3 \Rightarrow t_r(A) - 2t_r(B) = 3$ .....(2)

$\Rightarrow t_r(A) - 2[1 - 2t_r(A)] = 3$

$\Rightarrow t_r(A) = 1,\ t_r(B) = -1$

$\therefore\ t_r(A) - t_r(B) = 2$

17. Let $y = \left(3\sqrt{2}\right)x - 1$ be given line then find equation of straight line passes through A(1, 3) and makes an angle $\tan^{-1}\sqrt{2}$ with given line

(1) $5y + 4\sqrt{2}x = 4\sqrt{2} + 15$ (2) $5y - 4\sqrt{2}x = 4\sqrt{2} + 15$

(3) $4\sqrt{2}x - 5y = 4\sqrt{2} - 15$ (4) None of these

Ans. (1)

Sol. $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\sqrt{2} = \left|\dfrac{m_1 - 3\sqrt{2}}{1 + 3\sqrt{2}m_1}\right|$

$\Rightarrow \pm\sqrt{2}(1 + 3\sqrt{2}m_1) = (m_1 - 3\sqrt{2})$

$\sqrt{2} + 6m_1 = m_1 - 3\sqrt{2}$ [for positive sign]

$m_1 = \dfrac{-4\sqrt{2}}{5}$

$-\sqrt{2} - 6m_1 = m_1 - 3\sqrt{2}$

$2\sqrt{2} = 7m_1 \Rightarrow m_1 = \dfrac{2\sqrt{2}}{7}$

18. Find number of times, the digit '3' appears while writing integers from 1 to 1000.

Ans. 300

Sol. $\underline{3}\ \_\ \_ = (9 \times 9) \times 1$

$-\underline{3}\ \_ = (9 \times 9) \times 1$

$\_\ \_\ \underline{3} = (9 \times 9) \times 1$

$\underline{3}\ \underline{3}\ \_ = (9) \times 2$

$\underline{3}\ \_\ \underline{3} = (9) \times 2$

$-\ \underline{3}\ \underline{3} = (9) \times 2$

$\underline{3}\ \underline{3}\ \underline{3} = (1) \times 3$

total way = 243 + 54 + 3 = 300

19. Let P_1 be the plane $x - 2y + 2z - 3 = 0$ and A(1, 2, 3) lie on it. Let there be another plane P_2 which is parallel to P_1 and at unit distance from A. If P_2 is $ax + by + cz + d =$ the find positive value of $\left(\frac{b-d}{c-a}\right)$

Ans. 4

Sol. Let $P_2 : x - 2y + 2z + \lambda = 0$

$$\left|\frac{1-4+6+\lambda}{\sqrt{1+4+4}}\right| = 1$$

$|\lambda + 3| = 3$

$(\lambda + 3) = \pm 3$

$\lambda = -3 \pm 3$

$\lambda = 0$ or -6

$P_2 : x - 2y + 2z + 0 = 0$

$x - 2y + 2z - 6 = 0$

$$\frac{b-d}{c-a} = \frac{(-2)-(0 \text{ or } -6)}{(2-1)} = -2 \text{ or } 4$$

so positive is 4

20. If $f(x^2) + g(4 - x) = 4x^3$ and $g(x) + g(4 - x) = 0$, then the value of $\int_{-4}^{4} f(x^2)\,dx$

Ans. 512

Sol. $I = 2\int_0^4 f(x^2)\,dx$ ....(i)

$\Rightarrow\ I = 2\int_0^4 f((4-x)^2)\,dx$ ....(ii)

Adding equation (i) & (ii)

$2I = 2\int_0^4 \left[f(x^2) + f(4-x)^2\right] dx$ ....(iii)

Now using $f(x^2) + g(4 - x) = 4x^3$ ....(iv)

$x \to 4 - x$

$f((4 - x)^2) + g(x) = 4(4 - x)^3$ ....(v)

Adding equation (iv) & (v)

$f(x^2) + f(4 - x^2) + g(x) + g(4 - x) = 4(x^3 + (4 - x)^3]$

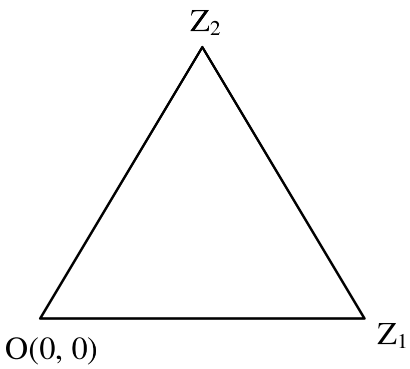
$\Rightarrow\ f(x^2) + f(4 - x^2) = 4\,[x^3 + (4 - x)^3]$

Now, $I = 4\int_0^4 (x^3 + (4-x)^3)\,dx = 512$

21. If roots of $x^2 + ax + 12 = 0$ and origin form equilateral triangle in complex plane, find $|a|$ equal to

Ans. 6

Sol.



for equilateral triangle

$Z_1^2 + Z_2^2 + O^2 = Z_1Z_2 + 0 + 0$

$(Z_1 + Z_2)^2 = 3Z_1Z_2$

$\Rightarrow (-a^2) = 3(12)$

$\Rightarrow a^2 = 36$

$a = -6$ or 6

$|a| = 6$

22. Average age of 25 teachers of a school is 40. If a teacher of age 60 year leave the school and a new teacher of 'x' age joined and new mean of their ages is 39 then find the value of 'x'

Ans. 35

Sol. $\bar{x} = \dfrac{\sum_{i=1}^{25} x_i}{25} = 40$

$\sum_{i=1}^{25} x_i = 1000$

when a teacher left school

$\sum_{i=1}^{24} x_i = 1000 - 60 = 940$

$\bar{x}' = \dfrac{\sum_{i=1}^{24} (x_i) + x}{25}$

$39 \times 25 = 940 + x$

$x = 35$

23. If $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{x^3}=\ell$ then

find $6\ell + 1$.

Ans. 4

Sol. $\ell=\lim\limits_{x\to 0}\dfrac{\left(x+\dfrac{x^3}{6}+....\right)-\left(x-\dfrac{x^3}{3}....\right)}{x^3}$

$\therefore\ \ell=\dfrac{1}{6}+\dfrac{1}{3}=\dfrac{1}{2}$

$\therefore\ 6\ell=4$

24. Find number of solutions of equation

$|\cot x|=\cot x+\dfrac{1}{\sin x}$ in $x\in[0,2\pi]$

Ans. 1

Sol. Case I : $x\in\left[0,\dfrac{\pi}{2}\right]\cup\left[\pi,\dfrac{3\pi}{2}\right]$

$\cot x=\cot x+\dfrac{1}{\sin x}\quad\Rightarrow$ not possible

Case : II $x\in\left[\dfrac{\pi}{2},\pi\right]\cup\left[\dfrac{3\pi}{2},2\pi\right]$

$-\cot x=\cot x+\dfrac{1}{\sin x}\ \Rightarrow\ \dfrac{-2\cos x}{\sin x}=\dfrac{1}{\sin x}$

$\Rightarrow \cos x=\dfrac{-1}{2}\ \Rightarrow x=\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

$= 1$

25. If $\displaystyle\int\frac{5x^8+7x^6}{(2x^7+x^2+1)^2}dx = f(x) + c$ and if $f(0) = 0$ & $f(1) = \dfrac{1}{k}$, then find k.

Ans. 4

Sol. $\displaystyle\int\frac{5x^8+7x^6}{(2x^7+x^2+1)^2}dx=\int\frac{5x^8+7x^6}{x^{14}\left(2+\dfrac{1}{x^5}+\dfrac{1}{x^7}\right)^2}dx$

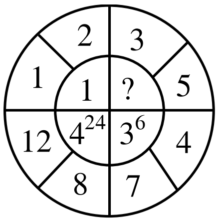
$$= \int \frac{\frac{5}{x^6}+\frac{7}{x^8}}{\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)} dx \ : \text{put } 2+\frac{1}{x^5}+\frac{1}{x^7}=t$$

$$= \int \frac{-dt}{t^2} = \frac{1}{t}+c \Rightarrow -\left(\frac{5}{x^6}+\frac{7}{x^8}\right)dx = dt$$

$$= f(x) = \frac{1}{2+\frac{1}{x^5}+\frac{1}{x^7}} = \frac{x^7}{2x^7+1+x^2}$$

$$f(x) = \frac{1}{4} = \frac{1}{k} \Rightarrow k = 4$$

26. Find the missing term in



Ans. 16

Sol. 4^{24} has base 4 (= 12 – 8)

3^6 has base 3 (= 7 – 4)

(?) will have base 2 (= 5 – 3)

Power 24 = 6×4 = (no. of divisor of 12) × (no. of divisor of 8)

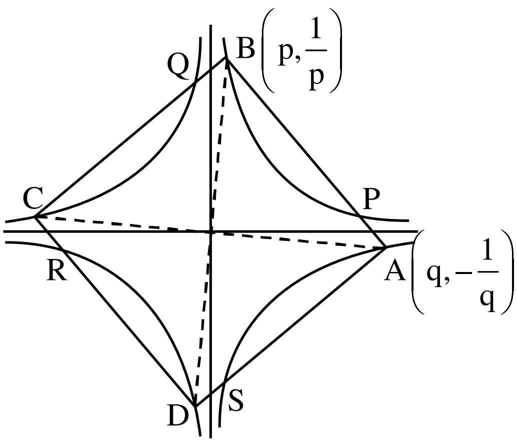
Power 6 = 2×3 = (no. of divisor of 7) × (no. of divisor of 4)

(?) will have power = (no. of divisor of 3) × (no. of divisor of 5) = $2 \times 2 = 4$

27. Four points lying on the curve $x^2y^2 = 1$ form a square such that mid-points of sides also lie on the given curve. Find the square of area of the square

Ans. 80

Sol.



$OA \perp OB$

$\Rightarrow \left(\frac{1}{p^2}\right)\left(-\frac{1}{q^2}\right) = -1$

$\Rightarrow p^2q^2 = 1$

$P\left(\frac{p+q}{2}, \frac{\frac{1}{p}-\frac{1}{q}}{2}\right)$ lies

on $x^2y^2 = 1$

$\Rightarrow (p+q)^2\left(\frac{1}{p}-\frac{1}{q}\right)^2 = 16$

$\Rightarrow (p+q)^2(p-q)^2 = 16$

$\Rightarrow (p^2-q^2)^2 = 16$

$\Rightarrow p^2 - \frac{1}{p^2} = \pm 4$

$\Rightarrow p^4 \pm 4p^2 - 1 = 0$

$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$

$\Rightarrow p^2 = 2 + \sqrt{5}$ or $-2 + \sqrt{5}$

$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2+\sqrt{5}}$ or $-2 + \sqrt{5} + \frac{1}{-2+\sqrt{5}} = 2\sqrt{5}$

$\text{Area} = 4\left(\frac{1}{2}\right)(OA)(OB) = 2(OB)^2 = 4\sqrt{5}$