

1. A triangle ABC in which side AB,BC,CA consist 5,3,6 points respectively, then the number of triangles that can be formed by these points are

- (1) 360                      (2) 333                      (3) 396                      (4) 320

Ans. (2)

Sol. Number of triangles =  ${}^{14}C_3 - {}^5C_3 - {}^3C_3 - {}^6C_3 = 333$

2. If  $(p \wedge q) \otimes (p \oplus q)$  is tautology, then

- (1)  $\otimes$  is  $\rightarrow$  and  $\oplus$  is  $\vee$                       (2)  $\otimes$  is  $\wedge$  and  $\oplus$  is  $\wedge$   
 (3)  $\otimes$  is  $\vee$  and  $\oplus$  is  $\vee$                       (4)  $\otimes$  is  $\vee$  and  $\oplus$  is  $\wedge$

Ans. (1)

p	q	$p \wedge r$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Sol.

3. The value of  $\lim_{n \rightarrow \infty} \frac{[r] + [2r] + [3r] + \dots + [nr]}{n^2}$  is (where  $[.]$  represents greatest integer function)

- (1)  $\frac{r}{2}$                       (2)  $\frac{r+1}{2}$                       (3)  $2r$                       (4)  $0$

Ans. (1)

Sol.  $r - 1 < [r] \leq r$

$$2r - 1 < [2r] \leq 2r$$

$\vdots$

$$nr - 1 < [nr] \leq nr$$

on adding

$$\frac{(r + 2r + \dots + nr) - n}{n^2} < \frac{[r] + [2r] + \dots + [nr]}{n^2} \leq \frac{r + 2r + \dots + nr}{n^2}$$

$$\begin{matrix} \downarrow & & \downarrow & & \downarrow \\ h(r) & & f(r) & & g(r) \end{matrix}$$

$$\lim_{n \rightarrow \infty} g(r) = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)r}{2}}{n^2} = \frac{r}{2}$$

$$\lim_{n \rightarrow \infty} h(r) = \lim_{n \rightarrow \infty} \frac{\frac{n(n+1)r}{2} - n}{n^2} = \frac{r}{2}$$

now by sandwich theorem

$$\lim_{n \rightarrow \infty} f(r) = \frac{r}{2}$$

4. The tangent at the point P(6,2) to the parabola  $y^2 = 4x - 20$  is also tangent to the ellipse  $\frac{x^2}{9} + \frac{y^2}{b} = 1$ . Then the value of 'b' is :

- (1) 1                                      (2) 7                                      (3) 6                                      (4) 2

**Ans.** (2)

**Sol.** T :  $2y = 2(x + 6) - 20 \Rightarrow y = x - 4$   
 $\therefore 16 = 9(1) + b \Rightarrow b = 7$

5. If z is a complex number satisfying

A :  $|z - 5| \leq 1$

B :  $\text{Re}((1 - i)z) \geq 1$

C :  $\text{Im}(z) \geq 1$ , then  $n(A \cap B \cap C)$  is

- (1) 0                                      (2) 1                                      (3) 2                                      (4) infinite

**Ans.** (4)

**Sol.** Let  $z = x + iy$

A :  $(x - 5)^2 + y^2 \leq 1$  .....(i)

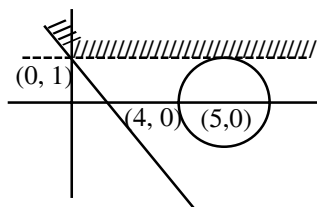
B :  $\text{Re}((1 - i)(x + iy)) \geq 1$

$\Rightarrow x + y \geq 1$  .....(ii)

C :  $\text{Im}(z) \geq 1$

$\Rightarrow y \geq 1$  .....(iii)

Plotting the regions given by (i), (ii) and (iii)



$\therefore n(A \cap B \cap C)$  is infinite

6. If  $f(x) = e^{-x} \sin x$  and  $F(x) = \int_0^x f(t) dt$  then  $\int_0^1 (F'(x) + f(x)) e^x dx$  lies in the interval

- (1)  $\left(\frac{327}{360}, \frac{329}{360}\right)$       (2)  $\left(\frac{329}{360}, \frac{330}{360}\right)$       (3)  $\left(\frac{330}{360}, \frac{331}{360}\right)$       (4)  $\left(\frac{331}{360}, \frac{332}{360}\right)$

**Ans.** (3)

**Sol.**  $F'(x) = f(x)$  by Leibnitz theorem  $\int_0^1 (F'(x) + f(x)) e^x dx = \int_0^1 2f(x) e^x dx$

$$I = \int_0^1 2 \sin x dx$$

$$I = 2(1 - \cos 1)$$

$$= \left\{ 1 - \left( 1 - \frac{1^2}{2!} + \frac{1^4}{4!} - \frac{1^6}{6!} + \dots \right) \right\}$$

$$2 \left\{ 1 - \left( 1 - \frac{1}{2} + \frac{1}{24} \right) \right\} < 2(1 - \cos 1) < 2 \left\{ 1 - \left( 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} \right) \right\}$$

$$\frac{330}{360} < 2(1 - \cos 1) < \frac{331}{360}$$

$$\frac{330}{360} < I < \frac{331}{360}$$

7. The value of  $\sum_{r=0}^6 {}^6C_r {}^6C_{6-r}$  is :

- (1) 924                      (2) 824                      (3) 972                      (4) 872

**Ans.** (1)

**Sol.**  $\sum_{r=0}^6 {}^6C_r {}^6C_{6-r} = {}^{12}C_6 = 924$

8. If  $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$ , then  $\alpha + \beta + \gamma$  is equal to (where  $[.]$  denotes greatest integer function)

- (1) 10                      (2) 2                      (3) 0                      (4) 1

**Ans.** (3)

**Sol.**  $10 \int_0^1 \frac{[\sin 2\pi x]}{e^{\{x\}}} dx \Rightarrow 10 \left[ \int_0^{1/2} 0 dx + \int_{1/2}^1 \frac{-1}{e^x} dx \right]$

$$= -10 \left[ \frac{e^{-x}}{-1} \right]_{1/2}^1 = 10 [e^{-1} - e^{-1/2}]$$

$$= 10e^{-1} - 10e^{-1/2}$$

$$\Rightarrow \alpha = 10, \beta = -10, \gamma = 0$$

$$\Rightarrow \alpha + \beta + \gamma = 0$$

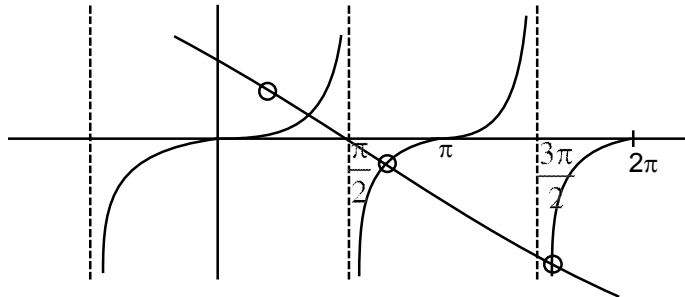
9. Number of solution of the equation  $x + 2 \tan x = \frac{\pi}{2}$  in  $x \in (0, 2\pi)$

- (1) 1                      (2) 2                      (3) 3                      (4) 0

**Ans.** (3)

**Sol.**  $x + 2 \tan x = \frac{\pi}{2}$

$$\tan x = -\frac{\pi}{2} + \frac{\pi}{4}$$



$\therefore$  3 solutions

**10.** If  $\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$  then number of values of  $x$  in  $[-1, 1]$  is/are (where  $[\cdot]$  is GIF)

(1) 0

(2) 1

(3) 2

(4) 3

**Ans.** (1)

**Sol.** Case-I:  $x \in \left[-1, -\sqrt{\frac{2}{3}}\right)$

$$\therefore \sin^{-1}(1) + \cos^{-1}(0) = x^2$$

$$\Rightarrow x = \pm\sqrt{\pi} \rightarrow \text{reject}$$

Case-II:  $x \in \left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

$$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$$

$$\Rightarrow x = \pm\sqrt{\pi} \rightarrow \text{Reject}$$

Case-III:  $x \in \left[\sqrt{\frac{2}{3}}, 1\right)$

$$\sin^{-1}(1) + \cos^{-1}(0) = x^2$$

$$x^2 = \pi \Rightarrow x = \pm\sqrt{\pi} \rightarrow \text{Reject}$$

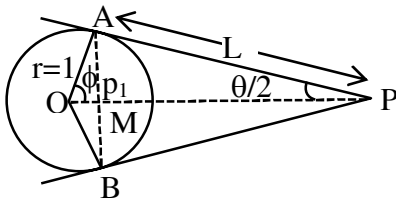
$\therefore$  no solution

**11.** If a circle  $x^2 + y^2 - 4x - 2y + 4 = 0$  from point P, tangents PA & PB are drawn to the given circle and angle between these tangents is  $\tan^{-1} \left( \frac{12}{5} \right)$ , then find  $\frac{\text{area (PAB)}}{\text{area(OAB)}}$  where (O is centre of circle)

- (1)  $\frac{9}{5}$                       (2)  $\frac{9}{4}$                       (3)  $\frac{3}{4}$                       (4)  $\frac{3}{2}$

**Ans.** (2)

**Sol.**  $\tan \theta = \frac{12}{5} = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} \Rightarrow \tan \frac{\theta}{2} = \frac{2}{3}$



$$OA = r = 1, \tan \frac{\theta}{2} = \frac{1}{L}$$

$$\frac{2}{3} = \frac{1}{L} \Rightarrow L = \frac{3}{2}$$

$$\phi = \left( \frac{\pi}{2} - \frac{\theta}{2} \right)$$

$$\tan \phi = \cot \frac{\theta}{2} = \frac{3}{2}$$

$$\sin \phi = \frac{2}{\sqrt{13}} = \frac{p_1}{1}$$

$$p_1 = \frac{2}{\sqrt{13}}$$

$$\text{Area of } \Delta OAM = \frac{1}{2} \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}} = \frac{3}{13}$$

$$\text{Area of } \Delta OAB = \frac{6}{13}$$

$$\text{Now Area of } \Delta PAB = rL - \text{ar}(\Delta OAB) = \frac{3}{2} - \frac{6}{13} = \frac{39-12}{26} = \frac{27}{26}$$

$$\text{Now } \frac{\text{Area } \Delta PAB}{\text{Area } \Delta OAB} = \frac{\frac{27}{26}}{\frac{6}{13}} = \frac{9}{4}$$

**12.** The value of  $\lim_{\theta \rightarrow 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$  is equal to

- (1)  $-\frac{1}{2}$                       (2) 0                      (3)  $\frac{1}{2}$                       (4)  $\frac{1}{4}$

**Ans.** (1)

**Sol.**  $\lim_{\theta \rightarrow 0} \frac{\tan(\pi - \pi \sin^2 \theta)}{\sin(2\pi \sin^2 \theta)} = \lim_{\theta \rightarrow 0} \frac{-\tan(\pi \sin^2 \theta)}{\sin(2\pi \sin^2 \theta)} = -\frac{1}{2}$

**13.** If  $f(x) = \begin{cases} \left(2 - \sin \frac{1}{x}\right) |x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$  then  $f(x)$  is

- (1) Monotonic in  $(-\infty, 0)$                       (2) Monotonic in  $(0, \infty)$   
 (3) Monotonic in  $(-\infty, 0) \cup (0, \infty)$                       (4) Non monotonic in  $(-\infty, 0) \cup (0, \infty)$

**Ans.** (4)

**Sol.**  $f(x) = \begin{cases} -\left(2 - \sin \frac{1}{x}\right)x, & x < 0 \\ 0, & x = 0 \\ \left(2 - \sin \frac{1}{x}\right)x, & x > 0 \end{cases}$

$$f'(x) = \begin{cases} -x \left(-\cos \frac{1}{x}\right) \left(-\frac{1}{x^2}\right) - \left(2 - \sin \frac{1}{x}\right), & x < 0 \\ x \left(-\cos \frac{1}{x}\right) \left(-\frac{1}{x^2}\right) + \left(2 - \sin \frac{1}{x}\right), & x > 0 \end{cases}$$

$$= \begin{cases} -\frac{1}{x} \cos \frac{1}{x} + \sin \frac{1}{x} - 2, & x < 0 \\ \frac{1}{x} \cos \frac{1}{x} - \sin \frac{1}{x} + 2, & x > 0 \end{cases}$$

**14.** If  $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$  and  $x, y, z$  are in A.P. with common difference  $d$ ,  $x \neq 3d$  then value of  $k^2$  is

- (1) 36                      (2) 72                      (3) 6                      (4)  $6\sqrt{2}$

**Ans.** (2)

**Sol.** 
$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_3 - 2R_2$

$$\begin{vmatrix} 0 & 4\sqrt{2} + k - 10\sqrt{2} & 0 \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$\Rightarrow (k - 6\sqrt{2})(4z - 5y) = 0$

$k = 6\sqrt{2} \quad \text{or} \quad 4z = 5y$

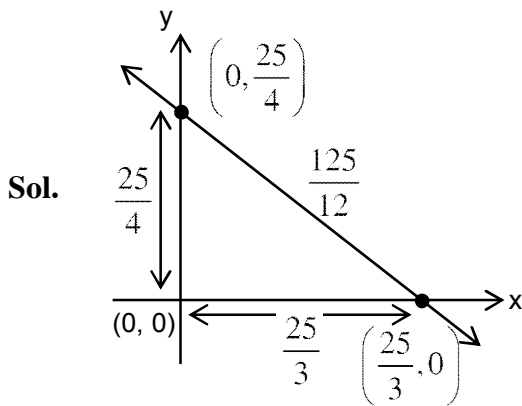
so  $k^2 = 72 \quad \Rightarrow x = 3d$

it is not possible

- 15.** Tangent at A(3, 4) of circle  $x^2 + y^2 = 25$  meets x and y axis at P and Q if a circle having centre as incentre of  $\Delta OPQ$  and passing through origin has radius r then  $r^2$  is

- (1)  $\frac{625}{72}$                       (2)  $\frac{625}{256}$                       (3)  $\frac{625}{64}$                       (4)  $\frac{625}{32}$

**Ans.** (1)



$T : 3x + 4y = 25$

$$I \equiv \left( \frac{\frac{625}{12}}{\frac{25}{4} + \frac{25}{3} + \frac{125}{12}}, \frac{\frac{625}{12}}{\frac{25}{4} + \frac{25}{3} + \frac{125}{12}} \right)$$

$\therefore I \equiv \left( \frac{625}{75+100+125}, \frac{625}{75+100+125} \right) \equiv \left( \frac{25}{12}, \frac{25}{12} \right)$

$\therefore r^2 = \left( \frac{25}{12} \right)^2 + \left( \frac{25}{12} \right)^2 = \frac{625}{72}$

**16.** If curve  $y(x)$  satisfied by differential equation  $2(x^2 + x^{5/4}) dy - y(x + x^{1/4}) dx = 2x^{9/4} dx$  and passing through  $\left(1, \frac{4}{3} - \ln 2\right)$ , then value of  $y(16)$  is

(1)  $\frac{128}{3} - \frac{16}{3} \ln 9 + \frac{4}{3} \ln 2$

(2)  $\frac{64}{3} - \frac{16}{3} \ln 9 + \frac{2}{3} \ln 2$

(3)  $\frac{128}{3} + \frac{16}{3} \ln 9 - \frac{4}{3} \ln 2$

(4)  $\frac{64}{3} + \frac{16}{3} \ln 9 - \frac{2}{3} \ln 2$

**Ans.** (1)

**Sol.**  $\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{5/4}}{(x + x^{1/4})}$

If  $= e^{\int -\frac{1}{2x} dx} = e^{-\frac{1}{2} \ln x} = \frac{1}{\sqrt{x}}$

Solution is  $\frac{y}{\sqrt{x}} = \int \frac{1}{\sqrt{x}} \frac{x^{5/4}}{(x + x^{1/4})} dx$

$\frac{y}{\sqrt{x}} = \int \frac{x^{3/4} + 1 - 1}{x^{1/4}(x^{3/4} + 1)} dx = \int \frac{1}{x^{1/4}} dx - \int \frac{1}{x^{1/4}(x^{3/4} + 1)} dx$

$\frac{y}{\sqrt{x}} = \frac{4x^{3/4}}{3} - \frac{4}{3} \ln(x^{3/4} + 1) + C$       {at  $x = 1, y = \frac{4}{3} - \ln 2$ }

$\frac{4}{3} - \ln 2 = \frac{4}{3} - \frac{4}{3} \ln 2 + C \Rightarrow \left(\frac{4}{3} - 1\right) \ln 2 = \frac{1}{3} \ln 2 = C$

at  $x = 16, \frac{y}{4} = \frac{4}{3} \cdot 8 - \frac{4}{3} \ln(9) + \frac{1}{3} \ln 2$

$y = \frac{128}{3} - \frac{16}{3} \ln 9 + \frac{4}{3} \ln 2$

**17.** If  $\cos x(3\sin x + \cos x + 3)dy = dx + y \sin x(3\sin x + \cos x + 3)dx$  then  $y\left(\frac{\pi}{3}\right)$  equals

(1)  $2 \ln\left(\frac{1+\sqrt{3}}{1+2\sqrt{3}}\right)$

(2)  $2 \ln\left(\frac{1+2\sqrt{3}}{1+\sqrt{3}}\right)$

(3)  $\ln\left(\frac{2\sqrt{3}-1}{\sqrt{3}+1}\right)$

(4)  $\ln\left(\frac{\sqrt{3}-1}{2\sqrt{3}+1}\right)$

**Ans.** (1)

**Sol.**  $(\cos x \cdot dy - \sin x \cdot y \cdot dx)(3\sin x + \cos x + 3) = dx$

$\Rightarrow d(y \cdot \cos x) = \frac{dx}{3\sin x + \cos x + 3}$

$\Rightarrow \int d(y \cdot \cos x) = \int \frac{\sec^2 \frac{x}{2} \cdot dx}{2 \tan^2 \frac{x}{2} + 6 \tan \frac{x}{2} + 4}$



$$\Rightarrow y \cdot \cos x = \int \frac{\frac{1}{2} \sec^2 \frac{x}{2} \cdot dx}{\tan^2 \frac{x}{2} + 3 \tan \frac{x}{2} + 2}$$

$$\Rightarrow y \cdot \cos x = \ln \left| \frac{\tan \frac{x}{2} + 1}{\tan \frac{x}{2} + 2} \right|$$

$$y \left( \frac{\pi}{3} \right) = 2 \ln \left( \frac{1 + \sqrt{3}}{1 + 2\sqrt{3}} \right)$$

- 18.** In binary input (having 0 and 1 as inputs) probability of 0 comes in even place is  $\frac{1}{2}$  and 0 comes in odd place is  $\frac{1}{3}$ . Find the probability that 01 is followed by 10.

(1)  $\frac{2}{9}$

(2)  $\frac{2}{3}$

(3)  $\frac{1}{3}$

(4)  $\frac{1}{9}$

**Ans.** (4)

**Sol.**

0	e	0	e
1	0	0	1

	odd	even
0	$\frac{1}{3}$	$\frac{1}{2}$
1	$\frac{2}{3}$	$\frac{1}{2}$

e	0	e	0
1	0	0	1

$$\begin{aligned} \text{req. probability} &= 2 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{9} \end{aligned}$$

- 19.** If image of point A(2, 3, 1) in the line  $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z+3}{-1}$  lies on the plane  $\alpha x + \beta y + \gamma z = 24$  also the line  $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-6}{15}$  lies in the plane then  $\alpha + \beta + \gamma$  is equal to

**Ans.** (19)

**Sol.** Let point on  $L_1$  :  $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-2}{-1}$  is

B  $(2\lambda + 1, \lambda + 4, -\lambda - 3)$

Now if B is foot of perpendicular of A in  $L_1$ , then  $AB \perp L_1$

$$2(2\lambda - 1) + 1(\lambda + 1) - (-\lambda - 4) = 0$$

$$6\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{2}$$

Hence B  $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

Now image A' (-2, 4, -6)

Now equation of plane containing A'(-2, 4, -6) and line  $L_2 : \frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-6}{15}$  is

$$\begin{vmatrix} x-1 & y-1 & z-6 \\ 1 & -2 & 15 \\ 3 & -3 & 12 \end{vmatrix} = 0$$

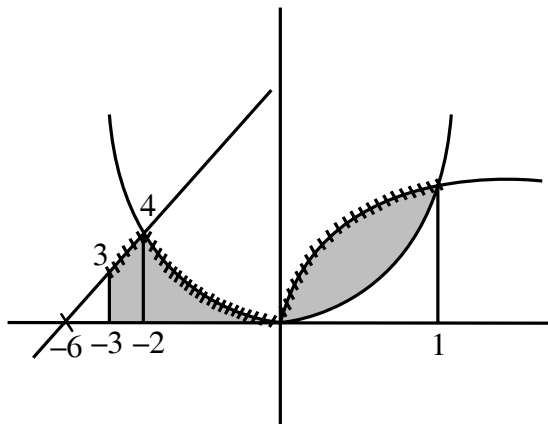
$$\Rightarrow 7x + 11y + z = 24$$

Hence  $\alpha = 7, \beta = 11, \gamma = 1$

- 20.** If area bounded by  $f(x) = \begin{cases} \min\{x+6, x^2\} & x \in [-3, 0] \\ \max\{x^2, \sqrt{x}\} & x \in [0, 1] \end{cases}$  and x-axis is A then find value of 6A

**Ans. 41**

**Sol.**



$$\text{area is } \int_{-3}^{-2} (x+6) dx + \int_{-2}^0 x^2 dx + \int_0^1 \sqrt{x} dx = A$$

$$= \frac{7}{2} + \left[ \frac{x^3}{3} \right]_{-2}^0 + \left[ \frac{2}{3} x^{3/2} \right]_0^1$$

$$= \frac{7}{2} + \frac{8}{3} + \frac{2}{3} = \frac{41}{6}$$

So,  $6A = 41$

**21.** Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and  $a + d = 2021$  also  $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ , (where  $\alpha, \beta \neq 0$ ),  $AB = B$  then  $ad - bc$  is equal to

**Ans.** 2022

**Sol.**  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$\begin{aligned} \therefore a\alpha + b\beta = \alpha & \left\{ \begin{aligned} \alpha(a-1) &= -b\beta \\ \& c\alpha + d\beta = \beta \end{aligned} \right. \\ & \left\{ \begin{aligned} \alpha(a-1) &= -b\beta \\ c\alpha &= \beta(1-d) \end{aligned} \right. \end{aligned}$$

$$\frac{a-1}{c} = \frac{b}{d-1}$$

$$ad - a - d + 1 = bc$$

$$\Rightarrow ad - bc = a + d + 1$$

$$\Rightarrow ad - bc = 2022$$

**22.** For  $3n$  observations of a ungrouped data. Variance is 4 and mean of first  $2n$  observation is 6 and mean of last  $n$  observation is 3, if 1 is added to first  $2n$  observations and 1 is subtracted to last  $n$  observations then variance of all  $3n$  observation is  $k$  then value of  $9k$  is :

**Ans.** 68

**Sol.** Let first  $2n$  observations are  $x_1, x_2, \dots, x_{2n}$  and last  $n$  observations are  $y_1, y_2, \dots, y_n$ .

$$\text{Now } \frac{\sum x_i}{2n} = 6, \frac{\sum y_i}{n} = 3 \Rightarrow \sum x_i = 12n, \sum y_i = 3n$$

$$\frac{\sum x_i + \sum y_i}{3n} = \frac{15n}{3n} = 5$$

$$\text{Now } \frac{\sum x_i^2 + \sum y_i^2}{3n} - 5^2 = 4$$

$$\Rightarrow \sum x_i^2 + \sum y_i^2 = 29 \times 3n = 87n$$

$$\text{Now mean is } \frac{\sum (x_i + 1) + \sum (y_i - 1)}{3n} = \frac{15n + 2n - n}{3n} = \frac{16}{3}$$

$$\text{Now variance is } \frac{\sum (x_i + 1)^2 + \sum (y_i - 1)^2}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{\sum x_i^2 + \sum y_i^2 + 2(\sum x_i - \sum y_i) + 3n}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 2(9n) + 3n}{3n} - \left(\frac{16}{3}\right)^2$$

$$= 29 + 6 + 1 - \left(\frac{16}{3}\right)^2 = \frac{324 - 256}{9} = \frac{68}{9} = k$$

$$\Rightarrow 9K = 68$$

**23.** Let  $f(x) = ax^2 + bx + c \forall x \in [-1, 1]$ ,  $f(-1) = 2$  and maximum value of  $f''(-1)$  is  $\frac{1}{2}$  and  $f'(-1) = 1$ ,

$f(x) \leq \alpha$  find  $\alpha_{\min}$

**Ans.** 5

**Sol.**  $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b, f''(x) = 2a$$

$$\text{given } f''(-1) = \frac{1}{2} \Rightarrow a = \frac{1}{4}$$

$$f'(-1) = 1 \Rightarrow b - 2a = 1 \Rightarrow b = \frac{3}{2}$$

$$f(-1) = a - b + c = 2 \Rightarrow c = \frac{13}{4}$$

$$\text{Now } f(x) = \frac{1}{4}(x^2 + 6x + 13), x \in [-1, 1]$$

$$f'(x) = \frac{1}{4}(2x + 6) = 0 \Rightarrow x = -3 \notin [-1, 1]$$

$$f(1) = 5, f(-1) = 2$$

$$f(x) \leq 5$$

$$\text{SO } \alpha_{\text{minimum}} = 5$$

**24.** If coefficient of third, fourth and fifth terms from beginning in the expansion of  $\left(x + \frac{a}{x^2}\right)^n$

( $n \in \mathbb{N}$ ) are in ratio 12 : 8 : 3 then the term independent of  $x$  is :

**Ans.** 4

**Sol.**  $T_{r+1} = {}^nC_r x^{n-r} \cdot \left(\frac{a}{x^2}\right)^r$

$$= {}^nC_r a^r x^{n-3r}$$

$$T_3 = {}^nC_2 a^2 x^{n-6}, T_4 = {}^nC_3 a^3 x^{n-9}$$

$$T_5 = {}^nC_4 a^4 x^{n-12}$$

$$\text{Now } \frac{\text{coefficient of } T_3}{\text{coefficient of } T_4} = \frac{{}^nC_2 \cdot a^2}{{}^nC_3 a^3} = \frac{3}{a(n-2)} = \frac{3}{2}$$

$$\Rightarrow a(n-2) = 2 \quad \text{(i)}$$

$$\text{and } \frac{\text{coefficient } T_4}{\text{coefficient } T_5} = \frac{{}^nC_3 \cdot a^3}{{}^nC_4 a^4} = \frac{4}{a(n-3)} = \frac{8}{3}$$

$$\Rightarrow a(n-3) = \frac{3}{2} \quad \text{(ii)}$$

$$\text{by (i) and (ii) } n = 6, a = \frac{1}{2}$$

for term independent of 'x'

$$n - 3r = 0 \Rightarrow r = \frac{n}{3} \Rightarrow r = \frac{6}{3} = 2$$

$$T_3 = {}^6C_2 \left(\frac{1}{2}\right)^2 \cdot x^0 = \frac{15}{4} = 3.75 \approx 4$$