MATHEMATICS

1. Number of solution of the equations $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5} = \sin^{-1} x$; $x \in [-1, 1]$ is

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (4)

Sol. Taking sine both sides

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}} = x$$

$\Rightarrow 3x\sqrt{25-16x^2} = 25x - 4x\sqrt{25-9x^2}$

$\Rightarrow x = 0$ or $3\sqrt{25-16x^2} = 25 - 4\sqrt{25-9x^2}$

$\Rightarrow 9(25 - 16x^2) = 625 - 200\sqrt{25-9x^2} + 16(25 - 9x^2)$

$\Rightarrow 200\sqrt{25-9x^2} = 800$

$\Rightarrow \sqrt{25-9x^2} = 4$

$\Rightarrow x^2 = 1$

$\Rightarrow x \pm 1$

$\therefore$ Total number of solution = 3

2.

Number of elements	Group-1	Group-2
Number of elements	10	n
Mean	2	3
variance	2	1

If combined variance of both groups is $\frac{17}{9}$, then find 'n'

Ans. (5)

Sol. For group-1 : $\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$

$\frac{\sum x_i^2}{10} - (2)^2 = 2 \Rightarrow \sum x_i^2 = 60$

for group-2 : $\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$

$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$

Now, combined variance

$$\sigma^2 = \frac{\sum(x_i^2+y_i^2)}{10+n} - \left(\frac{\sum(x_i+y_i)}{10+n}\right)^2$$

$$\Rightarrow \frac{17}{9} = \frac{60+10n}{10+n} - \frac{(20+3n)^2}{(10+n)^2}$$

$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$\Rightarrow 2n^2 - 5n - 25 = 0 \Rightarrow n = 5$

3. If $f(x) = \begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3} & , \quad x \neq 0 \\ \alpha & , \quad x = 0 \end{cases}$

then find α if f(x) is continuous at x = 0 (Here {x} denotes fractional part of x)

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{\sqrt{2}}$ (3) π (4) no value of α

Ans. (4)

Sol. $\text{RHL} = \lim_{x\to 0^+} \dfrac{\cos^{-1}(1-x^2)\sin^{1}(1-x)}{x(1-x^2)} = \dfrac{\pi}{2}\lim_{x\to 0^+} \dfrac{\cos^{-1}(1-x^2)}{x}$

$= \dfrac{\pi}{2}\lim_{x\to 0^+} \dfrac{-1}{\sqrt{1-(1-x^2)^2}}(-2x)$ (L′Hospital Rule)

$= \pi \lim_{x\to 0^+} \dfrac{x}{\sqrt{2x^2-x^4}} = \pi \lim_{x\to 0^+} \dfrac{1}{\sqrt{2-x^2}} = \dfrac{\pi}{\sqrt{2}}$

$\text{LHL} = \lim_{x\to 0^-} \dfrac{\cos^{-1}(1-(1+x)^2)\sin^{1}(-x)}{(1+x)-(1+x)^3} = \dfrac{\pi}{2}\lim_{x\to 0^-} \dfrac{\sin^{-1}x}{(1+x)[(1+x)^2-1]} = \dfrac{\pi}{2}\lim_{x\to 0^-} \dfrac{\sin^{-1}x}{x^2+2x}$

$= \dfrac{\pi}{2}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{4}$

As LHL $\neq$ RHL so f(x) is not continuous at x = 0

4. Let the circle $x^2 + y^2 - 9x - 2ay + c = 0$ has x–intercept and y–intercept respectively $2\sqrt{2}$ and $2\sqrt{5}$. Find distance of farthest tangent from origin which is perpendicular to $2y + x = 0$

(1) $\sqrt{6}$ (2) $\sqrt{30}$ (3) $\sqrt{10}$ (4) $2\sqrt{6}$

Ans. (1)

Sol. $2\sqrt{\dfrac{a^2}{4}-c} = 2\sqrt{2}$, $\qquad 2\sqrt{a^2-c} = 2\sqrt{5}$

$\dfrac{a^2}{4}-c = 2 \qquad a^2 - c = 5$

$c = \dfrac{a^2}{4}-2$

$a^2 = 4c + 8$

$3c + 8 = 5$

$c = -1, a^2 = 4$

Equation of tangent

$y = 2x + c'$

$D = \left|\dfrac{c'}{\sqrt{5}}\right| = r$

$\dfrac{c}{\sqrt{5}} = \sqrt{\dfrac{a^2}{4}+a^2-c}$

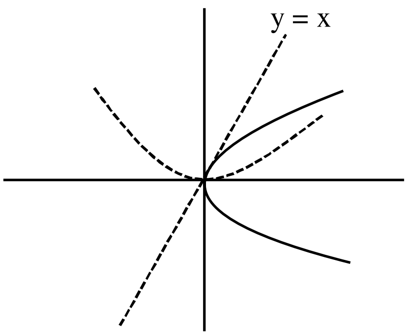
$\dfrac{c}{\sqrt{5}} = \sqrt{1+5} = \sqrt{30}$

$D = \sqrt{6}$

5. Let image of $y^2 = 4x$ in $y = x$ is a parabola. The equation of tangent from (2, 1) to new parabola is

(1) $y = x + 1$ (2) $x = y + 1$ (3) $y = 2x - 3$ (4) $y = \frac{x}{2} + 1$

Ans. (2)



Sol.

Image of $y^2 = 4x$ w.r.t. $y = x$ is $x^2 = 4y$

tangent from (2, 1)

$xx_1 = 2(y + y_1)$

$2x = 2(y + 1)$

$x = y + 1$

6. If point of intersection of the curves $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and $x^2+y^2=4b$, where $b>4$, lies on the curve $y^2 = 3x^2$ then find 'b'

(1) 4 (2) 12 (3) 10 (4) 6

Ans. (2)

Sol. $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ ......(1)

$x^2+y^2=4b$ ......(2)

$y^2 = 3x^2$ ......(3)

From eq (2) and (3) $x^2 = b$ and $y^2 = 3b$

from equation (1) $\frac{b}{16}+\frac{3b}{b^2}=1$

$\Rightarrow b^2 + 48 = 16b$

$\Rightarrow b = 12$

7. Find the area bounded between the curve whose differential equation are given by

$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ and $\frac{dx}{dy}=\frac{x^2-y^2}{2xy}$

Given that both the curve passes through the point (1, 1)

(1) $(\pi + 1)$ (2) $(\pi - 1)$ (3) $\left(\frac{\pi}{2}-1\right)$ (4) $\left(\frac{\pi}{2}+1\right)$

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put y = vx

$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$

$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} = -\frac{(v^2+1)}{2v}$

$\Rightarrow \frac{2v}{v^2+1}dv = -\frac{dx}{x}$

$\ell n(v^2 + 1) = -\ell nx + \ell nc \Rightarrow v^2 + 1 = \frac{c}{x}$

$\Rightarrow \frac{y^2}{x^2} + 1 = \frac{c}{x} \Rightarrow x^2 + y^2 = cx$

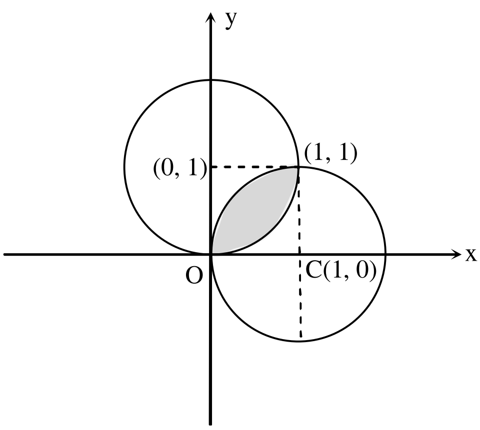
If pass through (1, 1)

$\therefore\ x^2 + y^2 - 2x = 0$

similarly for second differential equation $\frac{dx}{dy} = \frac{x^2 - y^2}{2xy}$

equation of curve is $x^2 + y^2 - 2y = 0$

Now required area is



$= \left(\frac{1}{4}\times\pi\times 1^2 - \frac{1}{2}\times 1\times 1\right)\times 2$

$= \left(\frac{\pi}{2} - 1\right)$ sq. units

8. If $\exp\left[\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\right)\log_e 2\right] \geq \log_{\sqrt{2}}|5\sqrt{7}+9i|$, then find the minimum value of $|z|$

(1) 3 (2) 2 (3) 5 (4) 6

Ans. (1)

Sol. $2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3 \quad \Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$

$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow |z|^2 - |z| - 6 \geq 0$

$(|z| - 3)(|z| + 2) \geq 0$

$|z|_{min} = 3$

9. Let f(x) ; $(0, \infty) \rightarrow R$ is defined $f(x + 1) = xf(x)$ & g(x) ; $R \rightarrow R$ is defined as $g(x) = \log(f(x))$, then the value of $|g''(5) - g''(1)|$ is

(1) $\frac{205}{144}$ (2) $\frac{144}{205}$ (3) $\frac{144}{113}$ (4) none of these

Ans. (1)

Sol. $g(x + 1) = \ell n(f(x + 1)) = \ell n(x f(x)) = \ell nx + \ell n(f(x))$

$\Rightarrow g(x + 1) - g(x) = \ell nx$

$g'(x + 1) - g'(x) = \frac{1}{x}$

$\Rightarrow g''(x + 1) - g''(x) = -\frac{1}{32}$ .....(i)

putting x = 1, 2, 3, 4 in (i), we get

$\Rightarrow g''(2) - g''(1) = -1$

$g''(3) - g''(2) = -\frac{1}{4}$

$g''(4) - g''(3) = -\frac{1}{9}$

$g''(5) - g''(4) = -\frac{1}{16}$

Adding,

$g''(5) - g''(1) = -1 - \frac{1}{4} - \frac{1}{9} - \frac{1}{16} = -\frac{205}{144}$

$\therefore |g''(5) - g''(1)| = \frac{205}{144}$

10. If y = y(x) the solution of the differential equation $\frac{dy}{dx} + y \tan x = \sin x$ and y(0) = 0, then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{1}{2}\ell n2$ (2) $\frac{1}{2\sqrt{2}}\ell n2$ (3) $\frac{1}{\sqrt{2}}\ell n2$ (4) $\ell n2$

Ans. (2)

Sol. Integrating factor = $e^{\int \tan x\, dx} = \sec x$

$\therefore$ solution of the equation is

$y \sec x = \int \sin x \times \sec x\ dx$

$\Rightarrow \frac{y}{\cos x} = \ell n(\sec x) + c$

put x = 0, c = 0

$\therefore\ y = \cos x\ \ell n\ (\sec x)$

put $x = \frac{\pi}{4}$

$y = \frac{1}{\sqrt{2}}\ell n\sqrt{2} = \frac{1}{2\sqrt{2}}\ell n2$

11. If (x) = $\begin{vmatrix} 1+\cos^2 x & \sin^2 x & \cos 2x \\ \cos^2 x & 1+\sin^2 x & \cos 2x \\ \cos^2 x & \sin^2 x & \sin 2x \end{vmatrix}$, then maximum value of f(x)

(1) $\sqrt{5}$ (2) 5 (3) $\sqrt{3}$ (4) $2\sqrt{5}$

Ans. (1)

Sol. $C_1 \rightarrow C_1 + C_2$

$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 2 & 1+\sin^2 x & \cos 2x \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$

$R_2 \rightarrow R_2 - R_1$

$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 0 & 1 & 0 \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$

f(x) = 2 sin 2x – cos 2x

$f(x)_{max} = \sqrt{5}$

12. The value of integral $\int_0^{10} \frac{[x].e^{[x]}}{e^{x-1}}dx$ is equal to

(1) 45 (1 – e) (2) 55 (e – 1) (3) 45 (e – 1) (4) 45 (e + 1)

Ans. (3)

Sol. $I = \int_0^{10} [x].e^{[x]+1-x}dx$

$= \int_1^2 e^{2-x}dx + \int_2^3 2.e^{3-x}dx + \int_3^4 3.e^{4-x}dx + ... + \int_9^{10} 9e^{10-x}dx$

$= -\{(1-e) + 2(1-e) + 3(1-e) + ... + 9(1-e)\}$

$= 45(e-1)$

13. If S = {0,1,2,....6} and a six digit and number is formed from it, find the probability it is divisible by '3'.

(1) $\frac{4}{5}$ (2) $\frac{4}{9}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. $6 \times 6 \times 5 \times 4 \times 3 \times 2$

$n(5) = 6 \times 6!$

0,1,2,3,4,5,6

$n(\in) = \frac{6! + 2\times5\times5!}{6\times6!}$

$= \frac{6+10}{36}$

$= \frac{4}{9}$

14. If two matrices A and B are $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix}$, then find k such that $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $(1 + k^2)\ b_2^2 \neq 2b_1b_2$.

Ans. 1

Sol. $\therefore$ A = XB

$\therefore \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\Rightarrow a_1 = \frac{1}{\sqrt{3}}(b_1 - b_2)$ and $a_2 = \frac{1}{\sqrt{3}}(kb_1 + b_2)$

Given that

$a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$\Rightarrow \frac{1}{3}\left(b_1 - b_2\right)^2 + \frac{1}{3}\left(kb_1 + b_2\right)^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$\Rightarrow (1 + k^2)\ b_1^2 + 2b_2^2 + 2\ (k-1)\ b_1b_2 = 2b_1^2 + 2b_2^2$

$\Rightarrow (1 + k^2)\ b_1^2 + 2(k-1)\ b_1b_2 = 2b_1^2$

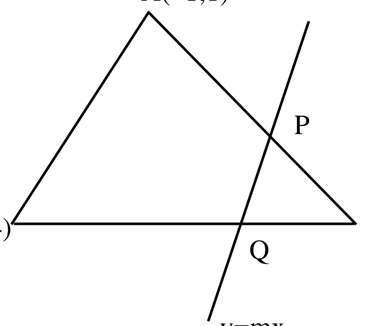
this is only possible when k = 1

15. Consider 3 points A(–1, 1), B(3, 4) and C (2, 0). The line y = mx cuts line AC and BC at points P and Q respectively. If area of $\Delta ABC = A_1$, and area of $\Delta PQC = A_2$ and $A_1 = 3A_2$, then positive value of m is

(1) 1 (2) $\frac{4}{15}$ (3) 2 (4) $\frac{15}{4}$

Ans. (1)

Sol.



$$A_1 = \Delta ABC = \frac{1}{2}\left\|\begin{matrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{matrix}\right\|$$

$$A_1 = \frac{13}{2}$$

Equation of line AC is $y - 1 = \frac{1}{3}(x + 1)$

solve it with line y = mx, we get $P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$

Equation of line BC is $y - 0 = 4(x - 2)$

Solve it with line y = mx, we get $Q\left(\frac{-8}{m-1}, \frac{-8m}{m-4}\right)$

$$A_2 = \text{Area of } \Delta PQC = \frac{1}{2}\left\|\begin{matrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{matrix}\right\| = \frac{A_1}{3} = \frac{13}{6}$$

$$\frac{1}{2}\left|m\left|\begin{matrix} 2 & 0 & 1 \\ 0 & \frac{2}{3m+1} & 1 \\ 0 & \frac{-8}{m-4} & 1 \end{matrix}\right|\right| = \frac{13}{6}$$

$$\frac{1}{2}\left|m(2)\left(\frac{2}{3m+1}+\frac{8}{m-4}\right)\right|=\frac{13}{6}$$

$$2m\left|\left(\frac{m-4+12m+4}{(3m+1)(m-4)}\right)\right|=\pm\frac{13}{6}$$

$$\frac{26m^2}{(3m+1)(m-4)}=\pm\frac{13}{6}$$

16. If $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + ...$ Also, $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$. then find a.

Ans. **(16)**

Sol. $S_n(x) = (\log_a x)\ \underbrace{2+3+6+11+......}_{7}$

$t_n = an^2 + bn + c$

$a + b + c = 2$ ...... (i)

$4a + 2b + c = 3$ ...... (ii)

$9a + 3b + c = 6$ ...... (iii)

$\Rightarrow a = 1; b = -2; c = 3$

$T_n = n^2 - 2n + 3 = (n-1)^2 + 2$

$\therefore S_{24}(x) = 1093 = \log_a x.(4372)$

$\Rightarrow x = a^{1/4}$ ...... (i)

Also, $S_{12}(2x) = 265 = (\log_a 2x).(530)$

$\Rightarrow 2x = a^{1/2}$ ...... (ii)

from (i) and (2)

$a^{1/2} = 2a^{1/4}$

$\Rightarrow a^2 = 16a \Rightarrow a = 16$

17. If $\frac{1}{16}$, a, b are in GP and $\frac{1}{a},\frac{1}{b},6$ are in AP, then

$72(a + b) =$

Ans. 14

Sol. $a^2=\frac{b}{16}$ and $\frac{2}{b}=\frac{1}{a}+6$

Solving, we get $a = \frac{1}{12}$ or $a = \frac{-1}{4}$ [rejected]

If $a = \frac{1}{12} \Rightarrow b = \frac{1}{9}$

$\therefore 72(a + b) = 72\left(\frac{1}{12}+\frac{1}{9}\right) = 14$

18. The number of points lying in the interior of line segments AB, BC, CD, DA of rectangle ABCD are 5, 6, 7 and 9. If α is the number of triangle made from the points by selecting each vertex from different line segments and β is the number of such quadrilaterals then the value of $|\alpha - \beta|$ is

(1) 711 (2) 717 (3) 919 (4) 926

Ans. (2)

Sol. $\alpha = {}^6C_1\,{}^7C_1{}^9C_1 + {}^5C_1\,{}^7C_1{}^9C_1 + {}^5C_1\,{}^6C_1{}^9C_1 + {}^5C_1\,{}^6C_1{}^7C_1 = 378 + 315 + 270 + 210 = 1173$

$\beta = {}^5C_1\,{}^6C_1{}^7C_1\,{}^9C_1 = 1890$

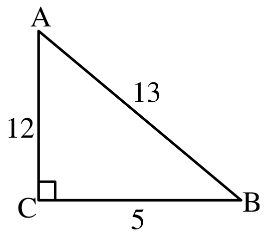
$\Rightarrow \beta - \alpha = 1890 - 1173 = 717$

19. Let a, b are sides of triangle where a = 5, b = 12 and area of triangle is 30, then value of 2R + r

(1) 13 (2) 15 (3) 20 (4) 17

Ans. (2)

Sol. $\angle C = 90^\circ$



$R = \frac{13}{2},\ S = \frac{12+5+13}{2} = 15$

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

$2R + r = 13 + 2 = 15$

20. Find the interval in which $f(x) = \log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}$ is increasing

(1) $(-\infty, -1) \cup [0, 1) \cup (1, \infty)$ (2) $(-1, -0) \cup [0, 1) \cup (1, \infty)$

(3) $(-1, -\infty)$ (4) $(-\infty, -1) \cup (-1, 1)$

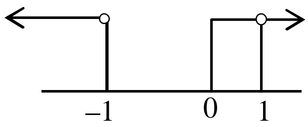
Ans. (1)

Sol. $f'(x) = \frac{x+1}{x-1} \times \frac{x+1-(x-1)}{(x+1)^2} + \frac{2}{(x-1)^2} = \frac{2}{(x-1)(x+1)} + \frac{2}{(x-1)^2}$

$= \frac{2}{(x-1)}\left(\frac{1}{x+1} + \frac{1}{x-1}\right) = \frac{2}{(x-1)^2(x+1)}\ (2x)$

$= \frac{4x}{(x-1)^2(x+1)}$

for increasing, $y'(x) \geq 0$



$\Rightarrow \dfrac{x}{(x-1)^2(x+1)} \geq 0$

$\Rightarrow x \in (-\infty, -1) \cup [0, 1) \cup (1, \infty)$

21. Consider the set A = {2, 3, 4, ..., 30}. A relation R defined on A × A is an equivalence relation such that (a, b) R (c, d) ⇒ ad = bc. If (c, d) = (3, 4). then find the number of ordered pair of (a, b)

Ans. (7)

Sol. (a, b) R (c, d) ⇒ ad = bc

$\therefore$ (a, b) R (3, 4) ⇒ 4a = 3b ⇒ $a = \dfrac{3}{4}b$

⇒ b is a multiple of 4

$\therefore$ (a, b) = (3, 4), (6, 8), (9, 12), (12, 16), (15, 20)(18, 24), (21, 28)

i.e., 7 ordered pairs

22. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and a vector $\vec{c}$ perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$ then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$

(1) 90 (2) –88 (3) 80 (4) 78

Ans. (2)

Sol. $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{c} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \lambda[i(1-4) - j(1-2) + k(2-1)]$$

$\vec{c} = \lambda(-3\hat{i} + \hat{j} + \hat{k})$

$\vec{c} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$

$\lambda(-3\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$

$\lambda(-3 - 1 + 3) = 8$

$-\lambda = 8$

$\lambda = -8$

$\vec{c} = -8(\vec{a} \times \vec{b})$

$\vec{c} \cdot (\vec{a} \times \vec{b}) = -8|\vec{a} \times \vec{b}|^2 = -8(\sqrt{9+1+1})^2 = -88$

23. Point (1, –1, 2) is the foot of perpendicular drawn from point (0, 3, 1) on the line $\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$. find the shortest distance between this line and the line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$.

(1) $\frac{61}{\sqrt{1314}}$ (2) $\frac{71}{\sqrt{1314}}$ (3) $\frac{91}{\sqrt{1314}}$ (4) $\frac{31}{\sqrt{1314}}$

Ans. (1)

Sol. Let A(0, 3, 1) and B(1, –1, 2)

d·r's of AB are 1, – 4, 1

$\ell(1) + 3(-4) + 4(1) = 0 \Rightarrow \ell = 8$

B line on $L_1 \Rightarrow \frac{1-a}{8} = -1 = \frac{2-b}{4}$

$\Rightarrow a = 9, b = 6$

$\Rightarrow L_1$ is $\frac{x-9}{8} = \frac{y-2}{3} = \frac{z-6}{4}$

L_2 is $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$

$$\text{Shortest distance} = \frac{\begin{vmatrix} 8 & 0 & 3 \\ 8 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\left|\left(8\hat{i}+3\hat{j}+4\hat{k}\right)\times\left(3\hat{i}+4\hat{j}+5\hat{k}\right)\right|}$$

$$= \frac{61}{\left|-\hat{i}-28\hat{j}+23\hat{k}\right|} = \frac{61}{\sqrt{1+784+529}} = \frac{61}{\sqrt{1314}}$$