MATHEMATICS

SCQ

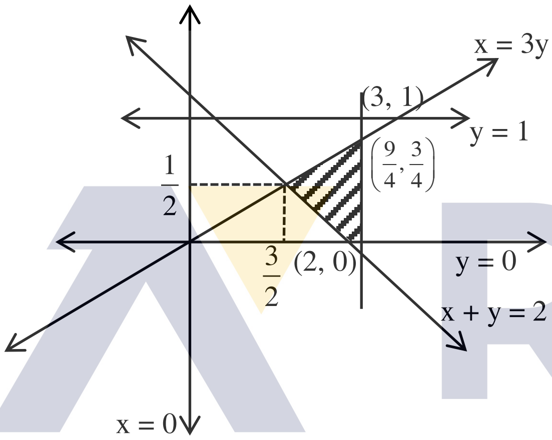
1. The area bounded by $0 \le x \le \frac{9}{4}, 0 \le y \le 1, x \ge 3y$ and $x + y \ge 2$ is

(A) $\frac{32}{11}$ (B) $\frac{11}{32}$ (C) $\frac{33}{5}$ (D) $\frac{37}{11}$

Ans. (B)

Sol. Given, $0 \le x \le \frac{9}{4}, 0 \le y \le 1, x \ge 3y$ and $x + y \le 2$

Area bounded by



$$\text{Area} = \frac{1}{2}\left(\frac{1}{2}+\frac{3}{4}\right) \times \frac{3}{4} - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{15}{32} - \frac{1}{8} = \frac{11}{32}$$

MCQ

2. A function f(x) is defined as $f : R \to R$ given by $f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$, then

(A) f(x) is increasing in (1, 2)

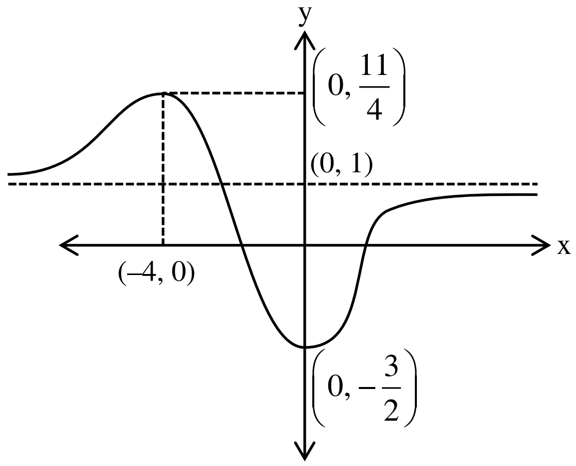
(B) f(x) is onto

(C) Range of f(x) is $\left[-\frac{3}{2}, 2\right]$

(D) f(x) is decreasing in (–2, –1)

Ans. (A, D)

Sol. $f'(x) = \dfrac{(x^2+2x+4)(2x-3)-(x^2-3x-6)(2x+2)}{(x^2+2x+4)^2} = \dfrac{5x(x+4)}{(x^2+2x+4)^2}$



Hence option (A) & (D)

Comprehension (3 & 4)

Three numbers are chosen randomly one after another with replacement from set $S = \{1, 2, 3, ..., 100\}$. Let P_1 be the probability that maximum of chosen number is atleast 81 and P_2 be the probability that minimum of chosen number of atmost 40, then

3. The value of $\frac{625}{4}P_1$ is

Ans. (76.25)

Sol. $P_1 = 1 - \frac{80}{100}\cdot\frac{80}{100}\cdot\frac{80}{100} = 1 - \frac{64}{125} = \frac{61}{125}$

$\Rightarrow 125p_1 = 61$

$\Rightarrow \frac{125p_1}{4} = \frac{61}{4} = 15.25$

$\Rightarrow \frac{625p_1}{4} = 76.25$

4. The value of $\frac{125}{4}P_2$ is

Ans. (24.50)

Sol. $p_2 = 1 - \frac{60}{100}\cdot\frac{60}{100}\cdot\frac{60}{100} = 1 - \frac{27}{125} = \frac{98}{125}$

$\Rightarrow \frac{125p_2}{4} = 24.50$

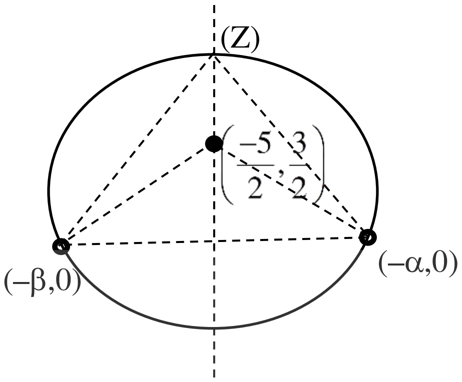
MCQ

5. For any complex number w = c + id, let arg (w) $\in (-\pi, \pi]$ where $i = \sqrt{-1}$. Let $\alpha, \beta \in R$ such that for all complex no. z = x + iy satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, then ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$, then

(A) $\beta = 4$ (B) $\alpha\beta = -4$ (C) $\alpha\beta = 4$ (D) $\alpha = -1$

Ans. (A,C)

Sol.



(i) $\frac{-\alpha-\beta}{2} = \frac{-5}{2} \Rightarrow \alpha + \beta = 5$ ....(i)

(ii) $\frac{\frac{3}{2}}{-\frac{5}{2}+\beta} \times \frac{\frac{3}{2}}{-\frac{5}{2}+\alpha} = -1 \quad \Rightarrow \frac{9}{4} = -\left(\alpha - \frac{5}{2}\right)\left(\beta - \frac{5}{2}\right) \quad \Rightarrow \frac{9}{4} = -\alpha\beta + \frac{5}{2}.5 - \frac{25}{4}$

$\Rightarrow \alpha\beta = \frac{25}{2} - \frac{9}{4} = 4$ ....(ii)

Comprehension (6 to 7)

The system of equation

x + 2y + 3z = α

4x + 5y + 6z = β

7x + 8y + 9z = γ – 1 is consistent for $\alpha, \beta, \gamma \in R$ and |M| represent the determinant of matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be plane contains all these (α, β, γ) for which the above system of linear equation is consistent. D be square of distance of point (0, 1, 0) from plane P, then

6. The value of |M| =

Ans. (1)

7. The value of D is =

Ans. (1.5)

Sol. **(6 & 7)**

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} \alpha & 2 & 3 \\ \beta & 5 & 6 \\ \gamma-1 & 8 & 9 \end{vmatrix} = -3(\alpha - 2\beta + \gamma - 1)$$

$$D_2 = \begin{vmatrix} 1 & \alpha & 3 \\ 4 & \beta & 6 \\ 7 & \gamma-1 & 9 \end{vmatrix} = 6(\alpha - 2\beta + \gamma - 1)$$

$$D_3 = \begin{vmatrix} 1 & 2 & \alpha \\ 4 & 5 & B \\ 7 & 8 & \gamma-1 \end{vmatrix} = -3(\alpha - 2\beta + \gamma - 1)$$

For consistent, $D_1 = D_2 = D_3 = 0$

$\Rightarrow \alpha - 2\beta + \gamma - 1 = 0$ ....(1)

6. $|M| = \alpha - 2\beta + \gamma = 1$

Plane contains (α,β,γ) **$x - 2y + z - 1 = 0$**

$\therefore \perp$ distance of $P(0,1,0) = \left|\dfrac{0-2+0-1}{\sqrt{6}}\right| = \dfrac{3}{\sqrt{6}}$

7. $D = \left(\frac{3}{\sqrt{6}}\right)^2 = \frac{9}{6} = \frac{3}{2} = 1.5$

Comprehension (8 to 9)

Let $L_1 = \sqrt{2}x + y - 1 = 0$, $L_2 = \sqrt{2}x - y + 1 = 0$

For a fixed constant λ let C be the locus of a point P such that product of distances of P from L_1 and L_2 is λ^2. The line $y = 2x + 1$ meets the C at 2 points R and S where the distance between R & S is $\sqrt{270}$. Let the $\perp$ bisector of RS meet curve C at 2 distinct point R′ & S′. Let D be square of distance between R′ & S′ , then

8. The value of λ^2 is

Ans. (9)

Sol. Let point p (h, k) Now

$$\left|\frac{\sqrt{2}h + k - 1}{\sqrt{3}}\right| \cdot \left|\frac{\sqrt{2}h - k + 1}{\sqrt{3}}\right| = \lambda^2$$

So lous is $|2x^2 - (y-1)^2| = 3\lambda^2$ ......(1)

Now line $y = 2x + 1$ cuts curve (1) at R & S So

$|2x^2 - (2x + 1 - 1)^2| = 3\lambda^2 \Rightarrow 2x^2 = 3\lambda^2$

Now $\left|2\left(\frac{y-1}{2}\right)^2 - (y-1)\right| = 3\lambda^2$ $2x^2 - 3\lambda^2 = 0$ ......(2)

$(y-1)^2 = \lambda^2$ ....(3) $\Rightarrow y^2 - 2y + 1 - 6\lambda^2 = 0$ ......(3)

Now distance set R & S is $\sqrt{270}$

$$\sqrt{(x_1 + x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2} = \sqrt{270}$$

$$0 - 4\left(\frac{-3\lambda^2}{2}\right) + 4 - 4(1 - 6\lambda^2) = 270$$

$6\lambda^2 + 4 - 4 + 24\lambda^2 = 270$

$30\lambda^2 = 270$

$\lambda^2 = 9$

9. The value of D^2 is

Ans. (77.14)

Sol. Now ⊥ bisector of R & S meets the curve at R' & S'

$y = -\frac{1}{2}x + 1$ cuts the curve C

$2x^2 - (y-1)^2 = 3d^2$ at R' & S'

$2x^2 - \frac{x^2}{4} = 3d^2 \Rightarrow \frac{7x^2}{4} = 3d^2$

and distance between P' and S' = $2\sqrt{x^2 + (y-1)^2} = 2\sqrt{x^2 + \frac{x^2}{4}} = 2\sqrt{\frac{5x^2}{4}} = 2\sqrt{\frac{5}{4} \cdot \frac{12\lambda^2}{7}}$

$R'S' = \sqrt{\frac{540}{7}}$

$D^2 = \frac{540}{7}$

SCQ

10. A triangle with two sides as y = 0 and x + y + 1 = 0 has its orthocentre at point (1, 1). Find equation of circumcircle of this triangle

(A) $x^2 + y^2 - 2x - 3y - 3 = 0$
(B) $x^2 + y^2 + 2x + 3y + 3 = 0$
(C) $x^2 + y^2 - 2x - 3y + 3 = 0$
(D) $x^2 + y^2 + x + y = 0$

Ans. (A)

Sol. Reflection of (1,1) about line y = 0 is (1, –1) = P

Reflection of (1,1) about line x + y + 1 = 0 is (–2, –2) = Q

Point of intersection of line y = 0 & x + y + 1 = 0 is (–1, 0) = C

Now ΔPQC is right angle triangle where right angled at C

⇒ equation of circumcircle of ΔPQC is same as equation of circumcircle of ΔABC which is

$(x-1)(x+2) + (y+1)(y+2) = 0$

$\Rightarrow x^2 + y^2 + x + 3y = 0$

MCQ

11. If $S_n = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right)$, then

(A) $\lim_{n\to\infty} \cot(S_n) = x$

(B) $S_{10}(x) = \frac{\pi}{2} - \cot^{-1}\left(\frac{10x}{1+11x^2}\right)$

(C) $S_3(x) = \tan^{-1}4x - \tan^{-1}x$

(D) $S_3(x) = \cot^{-1}\left(\frac{3x}{1+4x^2}\right)$

Ans. (A,B,C)

Sol. $\sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1+kx(k+1)x}\right)$

$\sum_{k=1}^{n} \tan^{-1}\left((k+1)x\right) - \tan^{-1}(kx)$

$S_n(x) = (\tan^{-1}(2x) - \tan^{-1}x) + (\tan^{-1}(3x) - \tan^{-1}(2x)) + \ldots\ldots\ldots\ldots + (\tan^{-1}(n+1)x + \tan^{-1}nx)$

$S_n(x) = \tan^{-1}(n+1)x - \tan^{-1}x$

(A) $\lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left(\frac{\pi}{2} - \tan^{-1}x\right) = x$

(B) $S_{10}(x) = \tan^{-1}(11x) - \tan^{-1}x = \tan^{-1}\left(\frac{11x-x}{1+11x^2}\right) = \tan^{-1}\left(\frac{10x}{1+11x^2}\right)$

$= \frac{\pi}{2} - \cot^{-1}\left(\frac{10x}{1+11x^2}\right)$

(C) $S_3(x) = \tan^{-1}(4x) - \tan^{-1}(x)$

SCQ

12. Let $\theta_1 + \theta_2 + \ldots. + \theta_{10} = 2\pi$ and $z_1 = e^{i\theta_1}$ and $z_n = z_{n-1}\, e^{i\theta_n}$.

S_1 : $|z_2 - z_1| + |z_3 - z_2| + \ldots.. + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$

S_2 : $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots.. + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$

then correct statement is

(A) Both S_1 and S_2 are true

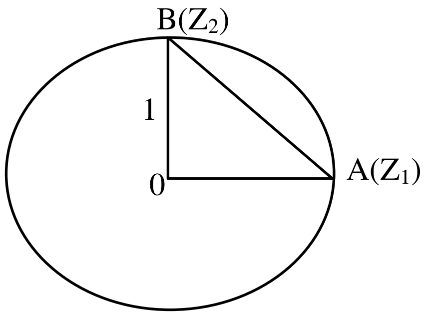
(B) S_1 is true and S_2 is false

(C) S_1 is false and S_2 is true

(D) Both S_1 and S_2 are false

Ans. (A)

Sol. $\frac{z_2}{z_1} = e^{i\theta_2}, \frac{z_3}{z_2} = e^{i\theta_3}$ **.........,** $\frac{z_{10}}{z_9} = e^{i\theta_{10}}$



Length of arc (AB) > AB $\quad \Rightarrow \left|\arg\left(\frac{z_2}{z_1}\right)\right| \geq |z_2 - z_1|$

Similarly $|Z_3 - Z_2| \leq \left|\arg\left(\frac{Z_3}{Z_2}\right)\right|$

$$|Z_1 - Z_{10}| \leq \left|\arg\left(\frac{Z_{10}}{Z_1}\right)\right|$$

Hence

$$|Z_2 - Z_1| + |Z_3 - Z_2| + ...+ |Z_1 - Z_{10}| \leq \left|\arg\frac{Z_2}{Z_1}\right| + \left|\arg\frac{Z_3}{Z_2}\right| +.....+ \left|\arg\frac{Z_1}{Z_{10}}\right|$$

$$\Rightarrow |Z_2 - Z_1| + |Z_3 - Z_2| +.....+ |Z_1 - Z_{10}| \leq \theta_2 + \theta_3 + ...+ \theta_{10} + 2\pi(\theta_2 + \theta_3 + ... + \theta_{10})$$

$$\Rightarrow |Z_2 - Z_1| + |Z_3 - Z_2| + ...+ |Z_1 - Z_{10}| \leq 2\pi$$

Now

$$|Z_2^2 - Z_1^2| = |Z_2 - Z_1||Z_2 + Z_1| \leq 2\left|\arg\left(\frac{Z_2}{Z_1}\right)\right|$$

Similarly , $|Z_3^2 - Z_2^2| \leq 2\left|\arg\left(\frac{Z_3}{Z_2}\right)\right|$

Hence, $|Z_2^2 - Z_1^2| + |Z_3^2 - Z_2^2| + ... + |Z_1^2 + Z_{10}^2| \leq 4\pi$

MCQ

13. Let E, F and G are square matrices of order 3 and I is the identity matrix of order 3. If G is the inverse of matrix (I – EF), then

(A) EFG = GEF

(B) $|FGE| \, . \, |I - FE| = |FE|$

(C) (I – FE) . (I + FGE) = I

(D) (I – FE) . (I – FGE) = I

Ans. (A,B,C)

Sol. (A) (I – EF)G = I = G (I – EFG)

$\Rightarrow G - EFG = I = G - GEF$

$\Rightarrow EFG = GEF$

(B) $G = (I - EF)^{-1}$

$\Rightarrow FGE = F(I - EF)^{-1} E$

$= (E^{-1} (I - EF)F^{-1})^{-1}$

$\Rightarrow FGE = (E^{-1}F^{-1} - I)^{-1}$

$\Rightarrow |FGE|.|E^{-1}F^{-1} - I| = 1$

$\Rightarrow |FGE|.|E^{-1}F^{-1} - I| \, |FE| = |FE|$

$\Rightarrow |FGE||I - FE| = |FE|$

(C) (I – FE).(I + FGE)

= I + FGE – FE – FE.FGE = 0

= I + FGE – FE – F (G – I)E

= I + FGE – FE – FGE + FE = I

MCQ

14. Let E, F, G be three events such that $P(E) = \frac{1}{8}$, $P(G) = \frac{1}{4}$, $P(F) = \frac{1}{6}$, $P(E \cap F \cap G) = \frac{1}{10}$, then which of the following is/are true?

(A) $P(E^C \cap F \cap G) \le \frac{1}{15}$

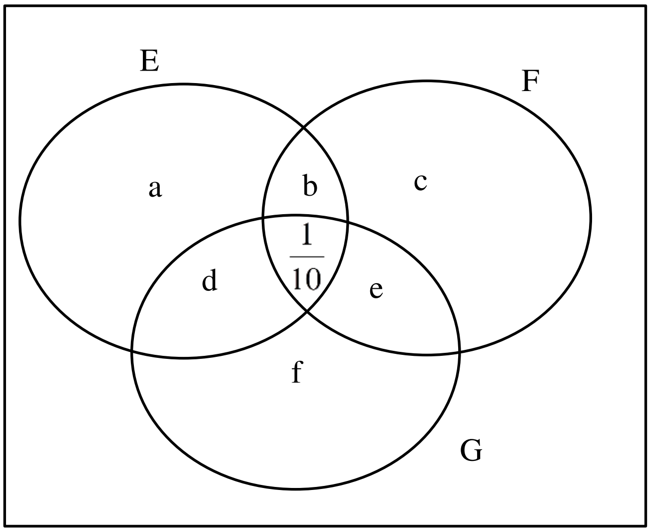
(B) $P(G^C \cap E \cap F) \le \frac{1}{40}$

(C) $P(E \cup G \cup F) \le \frac{13}{24}$

(D) $P(E^C \cap G^C \cap F^C) \le \frac{11}{24}$

Sol. (A, B, C)

Sol.



$$a + b + d + \frac{1}{10} = \frac{1}{8}$$

$$a + b + d = \frac{1}{40} \quad ....(1)$$

$$b + c + e + \frac{1}{10} = \frac{1}{6}$$

$$b + c + e = \frac{1}{15} \quad ....(2)$$

$$d + e + f + \frac{1}{10} = \frac{1}{4}$$

$$d + e + f = \frac{3}{20} \quad ....(3)$$

From (2) we get $e \le \frac{1}{15} \Rightarrow P(E^c \cap F \cap G) \le \frac{1}{15}$

From (1) we get $b \le \frac{1}{40} \Rightarrow P(G^c \cap F \cap E) \le \frac{1}{40}$

$\Rightarrow$ $P(E \cup G \cup F) \le \frac{1}{8} + \frac{1}{4} + \frac{1}{6} \le \frac{13}{24}$

$\Rightarrow$ $P(E^c \cap G^c \cap F^c) \ge 1 - \frac{13}{24} \ge \frac{11}{24}$

MCQ

15. In ΔPQR, p, q and r are sides opposites to the angles P, Q and R respectively. If $p < r$ and $p < q$, then

(A) $\cos P \geq 1 - \frac{p^2}{2qr}$

(B) $\frac{q+r}{p} > \frac{2\sqrt{\sin Q \sin R}}{\sin P}$

(C) $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

(D) $\cos R \geq \left(\frac{p-r}{p+q}\right)\cos P + \left(\frac{q-r}{p+q}\right)\cos Q$

Ans. (AD)

Sol. (A) $\cos P = \frac{q^2+r^2-p^2}{2qr} \geq \frac{2qr-p^2}{2qr} \geq 1-\frac{p^2}{2qr}$

(B) $\frac{q+r}{p} \geq \frac{2\sqrt{qr}}{p} \geq \frac{2\sqrt{\sin Q \sin R}}{\sin P}$

(C) option (c) in wrong when $\angle R = 90°$

(D) $p \cos R + q \cos R \geq p \cos P - r\cos P + q\cos Q - r\cos Q$

$p \cos R + r \cos P + q \cos R + r \cos Q \geq p \cos P + q \cos Q$

$q + p \geq p \cos P + q \cos Q$

MCQ

16. If $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}$ and $F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$ and Q is a 3×3 non singular matrix then

(A) $F = PEP$ and $P^2 = I$

(B) $|EQ|^3 > |EQ|^2$

(C) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(D) sum of diagonal elements of $(F + P^{-1}EP)$ = sum of diagonal elements of $(E + P^{-1}FP)$

Ans. (ACD)

Sol. (A) $P^2 = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = I$

$$PEP = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&2&3\\2&3&4\\8&13&18\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} = \begin{bmatrix}1&3&2\\8&18&13\\2&4&3\end{bmatrix} = F$$

$F = PEP \Rightarrow PF = P^2\,EP \Rightarrow PF = EP$

(B) $|EQ|^3 > |EQ|^2 \;\Rightarrow\; |E| = 0$ so it is false

(C) $|EQ + PFQ^{-1}| = |EQ + EPQ^{-1}| = |E\,(Q + PQ^{-1}| = |E|\,|Q + PQ^{-1}| = 0$

$|EQ| + |PFQ^{-1}| = 0 + 0 = 0$ so true

(D) $P = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix} \Rightarrow |P| = -1$

$$P^{-1} = \frac{\text{Adj } P}{|P|} = -\begin{bmatrix}-1&0&0\\0&0&-1\\0&-1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

$$P^{-1}EP = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&3&2\\2&4&3\\8&18&13\end{bmatrix}$$

$$= \begin{bmatrix}1&3&2\\8&18&13\\2&4&3\end{bmatrix} = F$$

$$F + P^{-1}EP = F + F = 2F = \begin{bmatrix}2&6&4\\16&36&26\\4&8&6\end{bmatrix}$$

Sum of Diagonal elements of $F + P^{-1}EP = 44$

$$E + P^{-1}FP = E + \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&3&2\\8&18&13\\2&4&3\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

$$= \begin{bmatrix}1&2&3\\2&3&4\\8&13&18\end{bmatrix} + \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&2&3\\8&13&18\\2&3&4\end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 8 \\ 16 & 26 & 36 \end{bmatrix}$$

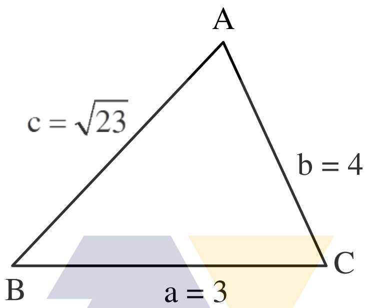
$\mathrm{Tr}\,(E + P^{-1}FP) = 44$

Integer

1. In ΔABC, If $AB = \sqrt{23}$, $BC = 3$, $AC = 4$, then find the value of $\dfrac{\cot A + \cot C}{\cot B}$.

Ans. 2

Sol.



$$\frac{\cot A + \cot C}{\cot B} = \frac{\dfrac{\cos A}{\sin A} + \dfrac{\cos C}{\sin C}}{\dfrac{\cos B}{\sin B}}$$

$$= \frac{\dfrac{b^2 + c^2 - a^2}{4\Delta} + \dfrac{a^2 + b^2 - c^2}{4\Delta}}{\dfrac{a^2 + c^2 - b^2}{4\Delta}} = \frac{2b^2}{a^2 + c^2 - b^2} = 2$$

2. Find the number of real roots of the equation $3x^2 - 4\,|x^2 - 1| + x - 1 = 0$

Ans. (4)

Sol. $3x^2 + x - 1 = 4\,|x^2 - 1|$

Case-I : $x \in [-1, 1]$

$3x^2 + 4x^2 - 4 + x - 1 = 0$

$\Rightarrow 7x^2 + x - 5 = 0$

$\Rightarrow x = \dfrac{-1 \pm \sqrt{141}}{14}$, $x \in [-1, 1]$

Both possible

Case-II : $x \in (-\infty, -1) \cup (1, \infty)$

$3x^2 - 4x^2 + 4 + x - 1 = 0$

$\Rightarrow x^2 - x - 3 = 0$

$\Rightarrow x = \frac{1+\sqrt{13}}{2}, \frac{1-\sqrt{13}}{2}, |x| \geq 1$

Both possible

$\therefore$ total number of solutions = 4

3. Let $\vec{u}, \vec{v}, \vec{w}$ be vectors in 3-D space where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other. Also $\vec{u}.\vec{w} = 1$, $\vec{v}.\vec{w} = 1$, $\vec{w}.\vec{w} = 4$. if volume of parallelepiped whose adjacent sides are represented by $\vec{u}$, $\vec{v}$ and $\vec{w}$ is $\sqrt{2}$, then $|3\vec{u} + 5\vec{v}| =$

Ans. (7)

Sol. $\left|\left[\vec{u}\,\vec{v}\,\vec{w}\right]\right| = \sqrt{2}$ $\quad \Rightarrow \left[\vec{u}\,\vec{v}\,\vec{w}\right]^2 = 2$ $\quad \Rightarrow \begin{vmatrix} \vec{u}.\vec{u} & \vec{u}.\vec{v} & \vec{u}.\vec{w} \\ \vec{v}.\vec{u} & \vec{v}.\vec{v} & \vec{v}.\vec{w} \\ \vec{w}.\vec{u} & \vec{w}.\vec{v} & \vec{w}.\vec{w} \end{vmatrix} = 2$

$\Rightarrow \begin{vmatrix} 1 & \vec{u}.\vec{v} & 1 \\ \vec{v}.\vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$ $\quad \Rightarrow 3 - \vec{u}.\vec{v}\,(4\vec{u}.\vec{v} - 1) + \vec{v}.\vec{u} - 1 = 2$

$\Rightarrow \vec{u}.\vec{v}\left(1 - 4\vec{u}.\vec{v} + 1\right) = 0$ $\quad \Rightarrow \vec{u}.\vec{v} = \frac{1}{2} \quad \left(\because\ \vec{u}.\vec{v} \neq 0\right)$

$\Rightarrow |3\vec{u} + 5\vec{v}| = \sqrt{9u^2 + 25v^2 + 30\vec{u}.\vec{v}} = 7$